Section 7.2 *Trigonometric integrals*

Learning outcomes

After completing this section, you will inshaAllah be able to

1. evaluate trigonometric integrals of the form
   a. $\int \sin^m x \cdot \cos^n x dx$
   b. $\int \tan^m x \cdot \sec^n x dx$
   c. $\int \sin mx \cdot \cos nx dx$, $\int \sin mx \cdot \sin nx dx$ and $\int \cos mx \cdot \cos nx dx$

Already know some methods of evaluating trigonometric integrals.

e.g.

- $\int \sqrt{\sin x} \cos x dx$ [change of variables]
- $\int \sin^n x dx$ or $\int \sec^n x dx$ [reduction formula]

In this section, we learn special methods for evaluating integrals involving trigonometric functions or products of trigonometric

Trigonometric Integrals CASE I:

Integrals of the form

$$\int \sin^m x \cdot \cos^n x\,dx$$

Can be divided in three classes

I(a) *m* positive odd integer, *n* any number

- Isolate the odd $\sin x$
- convert the rest into cosines
- substitute $u = \cos x$ and solve

See example 1, 2 done in class

I(b) *m* any number, n positive odd integer

- Isolate the odd $\cos x$
- convert the rest into sines
- substitute $u = \sin x$ and solve

See example 3, 4 done in class

I(c) Both *m* & n positive even integer

- Reduce to a combination of standard integrals, case(a), case(b) by using half angle formulas (for one or more times)

See example 5 done in class

- $\sin x \cos x = \frac{1}{2}\sin 2x$
- $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
- $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

Trigonometric Integrals CASE II:

Integrals of the form

$$\int \tan^m x \cdot \sec^n x\,dx$$

Can be divided in three classes

II(a) *m* any number, *n* even positive integer

See example 6 done in class

- Isolate $\sec^2 x$
- convert the rest into $\tan x$
- substitute $u = \tan x$ and solve

II(b) *m* positive odd integer, *n* any number

See example 7 done in class

- Isolate $\sec x \tan x$
- convert the rest into $\sec x$
- substitute $u = \sec x \tan x$ and solve

II(c) *m* positive even, *n* positive odd integer

See example 8 done in class

- Convert all in $\sec x$ and use reduction formula for $\int \sec^n x\,dx$

Trigonometric Integrals CASE III:

We can not use previous methods for such integrals if $m \neq n$

Integrals of the form

$$\int \sin mx \cdot \cos nx\,dx$$

$$\int \sin mx \cdot \sin nx\,dx$$

$$\int \cos mx \cdot \cos nx\,dx$$

for $m \neq n$

Use one of the following trigonometric formulas to bring the integral to a very simple form

1. $\sin mx \cos nx = \frac{1}{2}\left[\sin(m+n)x + \sin(m-n)x\right]$

2. $\cos mx \cos nx = \frac{1}{2}\left[\cos(m+n)x + \cos(m-n)x\right]$

3. $\sin mx \sin nx = \frac{1}{2}\left[\cos(m-n)x - \cos(m+n)x\right]$

See example 9 done in class

End of Section 7.2