Section 7.1 *Integration by parts*

Learning outcomes

After completing this section, you will inshaAllah be able to

1. learn the method of integration by parts
2. apply integration by parts to prove reduction formulas for integration of some functions
3. evaluate integrals using reduction formulas

Formula & techniques for integration by parts

If u and v are functions of x then

$$\int u \cdot v dx = u \int v dx - \int \left[u' \cdot \int v dx \right] dx$$

Most important step

Choice of 1st or 2nd function

We learn the by doing case by case examples

(1) Both functions are directly integrable but only one choice is useful — See example 1 done in class

(2) Only one of the two functions is directly integrable — See example 2 done in class

(3) There is only one function which is not directly integrable — See example 3 done in class

(4) Integrating by parts, for more than one time — See example 4 done in class

(5) "**I** + **I**" technique — See example 5 done in class

A combination of some of the above cases

Exercise: Evaluate

(a) $I = \int \sqrt{a^2 - x^2}\, dx$ [Case 2 + Case 5]

(b) $I = \int \cos(\ln x)\, dx$ [Case 2 + Case 5]

Reduction formulas (An application of integration by parts)

Reduction formula

A formula that expresses an integral involving a power of a function in terms of integral with smaller powers of the same function

Reduction formula for $\int \sin^n x\,dx \qquad (n \geq 1)$

$$\int \sin^n x\,dx = -\frac{\sin^{n-1} x \cdot \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx$$

See derivation in class

How to apply this reduction formula

- Reduces $\int \sin^n x\,dx$ to finding $\int \sin^{n-2} x\,dx$
- The **formula is applied iteratively**. Hence
  - for odd *n*, in the end we have to compute $\int \sin x\,dx$
  - for even *n*, in the end we have to compute $\int \sin^0 x\,dx$ or $\int dx$

See example 6 done in class

Reduction formulas (continued)

Reduction formula for $\int \cos^n x\,dx \quad (n \geq 1)$

$$\int \cos^n x\,dx = \frac{\cos^{n-1} x \cdot \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\,dx$$

Derivation similar to the formula done above

Reduction formula for $\int \sec^n x\,dx \quad (n \geq 2)$

$$\int \sec^n x\,dx = \frac{\sec^{n-2} x \cdot \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx$$

See derivation in class

Exercise Evaluate $\int \sec^4 x\,dx$

Evaluating Definite Integrals (using integration by parts)

Only trick

Put limits only after an integration has been completed

See example 7 done in class

End of Section 7.1