Learning outcomes

After completing this section, you will inshaAllah be able to

- 1. understand what is meant by representation of a function as power series
- 2. learn different methods of writing power series representation of a function using a known power series
- 3. use power series of $f(x) = \frac{1}{1-x}$ to make new power series representations

Basis of application of power series

is

representation of functions by power series

Meaning of representation of a function by power series

- We look at an example to understand.
- Recall from geometric series that

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$

for
$$|r| < 1$$
.

• Putting a = 1 and r = x implies

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

for
$$|x| < 1$$
.

The expression

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

for
$$|x| < 1$$

gives the power series representation of $f(x) = \frac{1}{1-x}$.

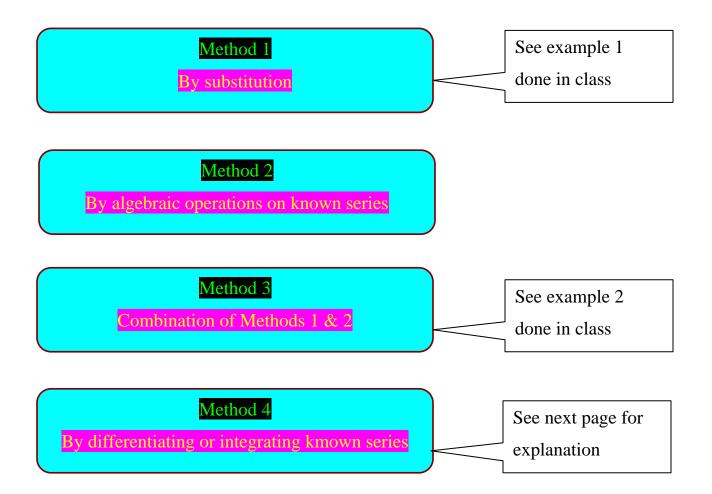
The interval of convergence is |x| < 1.

Methods for finding power series representation of a function (using known power series)

• Here we will only make new power series representations using

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \qquad \text{for } |x| < 1$$

• But the methods work in general.



Finding power series representation of functions

(by differentiating & integrating known power series representations)

Let $f(x) = \sum_{n=0}^{\infty} c_n (x - x_0)^n$ with radius of convergence R.

Then

$$f'(x) = \sum_{k=0}^{\infty} \frac{d}{dx} \left(c_k (x - x_0)^k \right)$$

and

$$\int f(x)dx = \sum_{k=0}^{\infty} \int \left(c_k(x-x_0)^k\right)dx.$$

Both have radius of convergence R.

See examples 3, 4 done in class

Note that radius of convergence stays the same but the interval of convergence may not be the same. There may be a difference at the end points of interval.