King Fahd Univ. of Petroleum and Minerals
Faculty of Sciences
Department of Mathematical Sciences

MAJOR No. 2
(MATH. 260-053 Sections 2 and 3)

Name:

ID:

Important instructions:
- Use an HB pencil or a pen (do not use red color)
- Solve the problems completely
- Write down your answers in a clear manner
- Justify all your steps
- Use the back of the page (verso) only for scratching

Prob. 1

Use the method of Gauss-Jordan to solve

$$\begin{cases} 2x_1 + 4x_2 - x_3 - 2x_4 + 2x_5 = 6 \\ x_1 + 3x_2 + 2x_3 - 7x_4 + 3x_5 = 9 \\ 5x_1 + 8x_2 - 7x_3 + 6x_4 + x_5 = 4 \end{cases}$$

Prob. 2

Find the inverse of the following matrix using the cofactors

$$\begin{pmatrix} +3 & 4 & -3 \\ +3 & 2 & -1 \\ -3 & 2 & -4 \end{pmatrix}$$

Prob. 3

Is

$$W = \left\{(x_1, x_2, x_3, x_4) \in \mathbf{R}^4 : \ x_1 x_2 = x_3 x_4\right\}$$

a subspace of $\mathbf{R}^4$? Justify your answer!

Prob. 4

Find a basis for the solution space of

$$\begin{cases} 3x_1 + x_2 - 3x_3 + 11x_4 + 10x_5 = 0 \\ 5x_1 + 8x_2 + 2x_3 - 2x_4 + 7x_5 = 0 \\ 2x_1 + 5x_2 - x_4 + 14x_5 = 0 \end{cases}$$

Prob. 5

Given the general solution

$$y(x) = e^x(C_1 e^{x\sqrt{2}} + C_2 e^{-x\sqrt{2}})$$

of a homogeneous second-order differential equation $ay'' + by' + cy = 0$ having constant coefficients, find this equation.

Prob. 6

Prove, using (02) two different methods, that the functions

$$f(x) \equiv 1,\ g(x) = x,\ h(x) = x^2$$

are linearly independent on the whole real line.

Prob. 7

Solve the initial value problem

$$\begin{cases} y^{(4)} = y^{(3)} + y'' + y' + 2y \\ y(0) = y'(0) = y''(0) = 0,\ y^{(3)}(0) = 30. \end{cases}$$