# King Fahd Univ. of Petroleum and Minerals
## Faculty of Sciences
## Department of Mathematical Sciences

FINAL EXAM
(MATH. 101-041 Sections 4 & 8)

**Name:**
**ID:**

**Prob. 1**
Find an approximate value for
(a) $(1.99)^5$
(b) $(8.02)^{-\frac{1}{3}}$

## Prob. 2
Find
a)
$$\sin(\tan^{-1} \frac{3}{5})$$

b)
$$\tan(\sin^{-1} \frac{7}{3})$$

## Prob. 3

A road is to be built between two cities $C_1$ and $C_2$, which are on opposite sides of a river of uniform width $r$. Because of the river, a bridge must be built. $C_1$ is $\alpha$ units from the river and $C_2$ is $\beta$ units from the river, $\alpha \leq \beta$. Where should the bridge be located in order to minimize the total distance between the cities?

**Prob. 4**

Prove that

$$\frac{d}{dx}\left[x\tan^{-1}x - \frac{1}{2}\ln(1+x^2)\right] = \tan^{-1}x$$

**Prob. 5**

Analyze and sketch the graph of the function

$$f(x) = (x - 1)^{\frac{1}{3}} - 1$$

### Prob. 6

A company begins a television advertising campaign to market a new product. The percentage of the "target market" that buys a product is a function of the length of the advertising campaign. The television station estimates this percentage as $1 - e^{-0.04t}$ for this type of product, where $t =$ the number of days of the campaign. The target market is estimated to be $1,000,000$ people and the price per unit is $0.50$ $SAR$. The costs of advertising are $1000$ $SAR$ per day. Find the length of the advertising campaign that will result in the maximum profit. (Hint: Profit = Revenue - Cost).

**Prob. 7**
Prove that
$$\frac{d}{dx}\left[\csc^{-1} x\right] = \frac{-1}{|x|\sqrt{x^2 - 1}}$$

**Prob. 8**

Analyze and sketch the graph of the function

$$g(x) = \frac{x^3 + 2x^2 - 15x}{x^2 - 5x - 14}$$

### Prob. 9

The police discover the body of a calculus professor. Critical to solving the crime is determining when the murder was committed. The police call the coroner, who arrives at 12 noon. He immediately takes the temperature of the body and finds it to be $94.6^0$. He waits one (1) hour, takes the temperature again and finds it to be $93.4^0$. He also notes that the temperature of the room is $70^0$. When was the murder committed? (The normal temperature of the body (alive) is $98.6^0$)

(Hint: The temperature of a cooling object drops at a rate that is proportional to the difference $T - C$ where $C$ is the constant temperature of the surrounding medium. Thus

$$\frac{dT}{dt} = -k(T - C) \tag{1}$$

The function that satisfies Eq.(1) is $T = T(t) = ae^{-kt} + C$.)

## Prob. 10

Analyze and sketch the graph of the function

$$h(t) = \left| \frac{1}{x} - 2 \right|$$