

King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Semester II, 2005-2006 (052)
MATH 101 — Final Exam

Code 001

Name: _____ ID: _____ Section: _____

Q1. $\lim_{x \rightarrow -\infty} \frac{x^3 - x^2 + 1}{2 - x - x^2} =$

- (a) 0
- (b) 1
- (c) -1
- (d) $-\infty$
- (e) $+\infty$

Q2. The area of the triangle formed by the tangent line to the curve $xy = 1$ at $x = -3$ and the coordinate axes is equal to

- (a) $5/3$
- (b) 2
- (c) $-1/9$
- (d) $7/3$
- (e) 3

Q3. The derivative $\frac{dy}{dx}$ of $y = \cosh^{-1} \sqrt{x^2 + 1}$ is equal to

- (a) $\frac{1}{\sqrt{x^2+1}}$
- (b) $\frac{1}{x\sqrt{x^2+1}}$
- (c) $\frac{x}{|x|\sqrt{x^2+1}}$
- (d) $\frac{-1}{\sqrt{x^2+1}}$
- (e) $\frac{1}{\sqrt{1-x^2}}$

Q4. The linear approximation of the function $f(x) = (1+x)^{1/5}$ at 0 is

- (a) $\frac{x}{5} + 1$
- (b) $\frac{x}{5} - 1$
- (c) $-\frac{x}{5} - 1$
- (d) $-\frac{x}{5} + 1$
- (e) $x + \frac{1}{5}$

Q5. If the derivative of a function f is differentiable for all x , then

$$\lim_{x \rightarrow 1} \frac{\lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h/2} - 2f'(x)}{\frac{x}{2} - \frac{1}{2}} =$$

- (a) $-f'(1)$
- (b) $2f''(\frac{x}{2})$
- (c) $\frac{f''(1)}{4}$
- (d) $-\frac{1}{2}f'(\frac{x}{2})$
- (e) $-4f''(1)$

Q6. The set of x -coordinates of the points at which the function $f(x) = \frac{x-1}{x^4-2x^3-x+2}$ has removable discontinuity is:

- (a) $\{1\}$
- (b) $\{2\}$
- (c) $\{1, 2\}$
- (d) $\{-2, -1\}$
- (e) $\{-2, 1\}$

Q7. Regarding y as the dependent variable and x as the independent variable in the equation $x^y = y^x$, the derivative $\frac{dy}{dx}$ is equal to

- (a) $\frac{yx^{y-1}}{xy^{x-1}}$
- (b) $\frac{xy^{x-1}}{yx^{y-1}}$
- (c) $\frac{x \ln y - y}{y \ln x - x}$
- (d) $\frac{y^2 - xy \ln y}{x^2 - xy \ln x}$
- (e) $\frac{\ln y - \frac{y}{x}}{\ln x}$

Q8. The function $f(x) = -\frac{1}{4}x^4 + 2x^2 + 3$ has exactly

- (a) one local maximum and one local minimum
- (b) two local minimums and one local maximum
- (c) one local minimum and two local maximums
- (d) one local minimum and no local maximums
- (e) one local maximum and no local minimum

Q9. Using Newton's method with initial approximation $x_1 := -1$, the second approximation x_2 of the root of the equation $5x^3 - 3x^2 + x + 2 = 0$ is:

- (a) $-\frac{15}{22}$
- (b) $-\frac{1}{2}$
- (c) $-\frac{10}{3}$
- (d) $-\frac{1}{4}$
- (e) $-\frac{1}{6}$

Q10. $\lim_{x \rightarrow 1^-} \frac{1-x^2}{x^3-1} =$

(a) $\frac{1}{3}$

(b) $-\frac{1}{3}$

(c) $\frac{2}{3}$

(d) $\frac{-2}{3}$

(e) ∞

Q11. If the normal line to the curve $y = f(x)$ at the point $(-1, 3)$ passes through the point $(0, 5)$, then $f'(-1) =$

(a) -2

(b) $3/2$

(c) $1/2$

(d) $-1/2$

(e) -1

Q12. The largest possible δ such that

$$|x - 1| < \delta \Rightarrow |x^2 - 1| < \frac{1}{2}$$

is:

(a) $\sqrt{\frac{3}{2}} - 1$

(b) $1 - \frac{1}{\sqrt{2}}$

(c) $\sqrt{\frac{3}{2}}$

(d) $\frac{1}{\sqrt{2}}$

(e) 1

Q13. If $f(x) = \frac{\sin x \sec x}{1+x \tan x}$, then $f'(0) =$

- (a) -2
- (b) -1
- (c) 0
- (d) 1
- (e) 2

Q14. $D^{1000}(xe^{-x})$ is equal to

- (a) $(x - 1001)e^{-x}$
- (b) $(1000 - x)e^{-x}$
- (c) $(999 - x)e^{-x}$
- (d) $(x - 1000)e^{-x}$
- (e) $(1001 - x)e^{-x}$

Q15. The equation $x^{201} + 804x = 9 - \frac{402}{101}x^{101}$ has

- (a) exactly one real solution
- (b) no real solutions
- (c) exactly 201 real solutions
- (d) three real solutions
- (e) one real solution in $(-\infty, 0)$ and another one in $(0, +\infty)$

Q16. A point of the curve of $y = \ln(\ln x)$ at which the tangent has slope $\frac{1}{e}$ is

- (a) $(\frac{1}{e}, 0)$
- (b) $(e^e, 1)$
- (c) $(e, 0)$
- (d) Does not have
- (e) $(e^2, \ln 2)$

Q17. The function $f(x) = \frac{x^2+5}{x+2}$

- (a) changes its concavity from concave down to concave up and has no inflection points
- (b) changes its concavity from concave down to concave up and has an inflection point at -2
- (c) changes its concavity from concave up to concave down and has no inflection points
- (d) changes its concavity from concave up to concave down and has an inflection point at -2
- (e) f is always concave up

Q18. A sketch of the curve of $f(x) = x^2e^{-x^2}$ would be

Q19. Which one of the following curves corresponds to the function

$$f(x) = \frac{x^5}{(x^2 - 1)^2}.$$

Q20. The cost of a printer is 625 Riyals, and its value is depreciating (i.e. decreasing) with time according to the formula

$$\frac{dV}{dt} = -800(2t + 1)^{-2},$$

where V Riyals is its value t years after its purchase. What is the value of the printer three years after its purchase (rounded to two decimals)?

- (a) 280.33 Riyals
- (b) 292.04 Riyals
- (c) 272.74 Riyals
- (d) 282.14 Riyals
- (e) 290.04 Riyals

Q21. The set of vertical asymptotes for the graph of

$$f(x) = \frac{x^4 - 1}{x^3 - x}$$

is:

- (a) $\{x = -1, x = 0, x = 1\}$
- (b) $\{x = -1, x = 0\}$
- (c) $\{x = 0, x = 1\}$
- (d) $\{x = -1, x = 1\}$
- (e) $\{x = 0\}$

Q22. The derivative $\frac{dy}{dx}$ for $y = \sqrt{x + \coth \sqrt{1 + x^2}}$ is:

- (a) $\frac{\sqrt{1+x^2} + \operatorname{csch}^2(\sqrt{1+x^2})}{2\sqrt{1+x^2}\sqrt{x+\coth(\sqrt{1+x^2})}}$
- (b) $\frac{1 - \operatorname{csch}^2(\sqrt{1+x^2})}{2\sqrt{x+\coth(\sqrt{1+x^2})}}$
- (c) $\frac{-x \cosh^2(\sqrt{1+x^2})}{2\sqrt{x+\coth(\sqrt{1+x^2})}}$
- (d) $\frac{1}{2\sqrt{x+\coth(\sqrt{1+x^2})}}$
- (e) $\frac{\sqrt{1+x^2} - x \operatorname{csch}^2(\sqrt{1+x^2})}{2\sqrt{1+x^2}\sqrt{x+\coth(\sqrt{1+x^2})}}$

Q23. The equation of the line through the point $(2, 3)$ that forms with the positive x -axis and the positive y -axis a triangle of non-zero least area is:

(a) $y = -\frac{2}{3}x + \frac{13}{3}$

(b) $y = -3x + 9$

(c) $y = -\frac{3}{2}x + 6$

(d) $y = -x + 5$

(e) $y = 2x - 1$

Q24. Which one of the following statements is always true?

(a) If f is continuous on $[-1, 1]$ and $f(-1) = 4$ and $f(1) = 3$, then there exists a number r such that $|r| < 1$ and $f(r) = \pi$

(b) If the line $x = 1$ is a vertical asymptote of $y = f(x)$, then f is not defined at 1

(c) If f is continuous at a , then f is differentiable at a

(d) If $f(x) > 1$ for all x and $\lim_{x \rightarrow 0} f(x)$ exists, then $\lim_{x \rightarrow 0} f(x) > 1$

(e) If $\lim_{x \rightarrow 6} f(x)g(x)$ exists, then it is equal to $f(6)g(6)$

Q25. Water is leaking out (escaping) of an inverted conical (cone shape) tank at a rate of $10^4 \text{ cm}^3/\text{min}$. At the same time water is being pumped into the tank at a constant rate. The height of the tank is $H = 8\text{ m}$ and its diameter at the top is $D = 6\text{ m}$. If the water level is rising at the rate of $\frac{64}{81\pi} \text{ cm}/\text{min}$ when the height h of the water is $h = 3\text{ m}$, the rate at which the water is being pumped into the tank is

(a) $20000 \text{ cm}^3/\text{min}$

(b) $10000 \text{ cm}^3/\text{min}$

(c) $15000 \text{ cm}^3/\text{min}$

(d) $10000 \cdot \frac{64}{81\pi} \text{ cm}^3/\text{min}$

(e) $(10000 + \frac{64000}{81\pi}) \text{ cm}^3/\text{min}$