

A Game Theoretic Model for Smart Grids Demand Management

Slim Belhaiza and Uthman Baroudi

Abstract—Demand-side management (DSM) plays a key role in the future of smart grids. Recently, DSM researchers have developed various mathematical models to optimize the demand response. Most of these works ignore the channel impairments' impact on the optimization process. In this paper, we propose a new noncooperative game theoretic model for the management of smart grid's demand considering the packet error rate in our formulation. We set the Nash equilibrium conditions for the proposed model. Under an assumption on the form of the utility functions, we develop a 0-1 mixed linear programming approach to compute nondominated extreme Nash equilibria. Results on a numerical example are provided and some useful insights are presented. Under some assumptions and a fully proven proposition, a feasible nondominated Nash equilibrium solution is found. Finally, we report and comment on computational experiments on randomly generated smart grid DSM game instances with different characteristics.

Index Terms—Advanced metering infrastructure (AMI), demand-side management (DSM), game theory, Nash equilibrium, smart grid.

I. INTRODUCTION

A SMART grid [1], [2] is an intelligent electric power infrastructure that collects information via modern communication technologies and in particular wireless networks to provide efficient, reliable, and cost-saving energy generation and distribution. A smart grid comprises five major facilities: 1) generation; 2) distributed generation; 3) transmission; 4) distribution; and 5) end user (home/building) [2], [3]. The ability to monitor and influence each user's usage in real time can enable distribution operators match supply with demand effectively and realize the potential of digital power [3].

“DSM refers to the tools and mechanisms that influence the customer's use of energy” [4]. The actions taken by demand-side management (DSM) can be classified into two broad categories: 1) reduce consumption; and 2) shift consumption. In the former, we intend to reduce the energy consumption of individual end customers, while the later focuses on varying the time of consumption such that the peak load is reduced. In fact, the specific response depends

on the metering capabilities deployed in the smart grid. With the deployment of advanced metering infrastructure (AMI), advanced demand responses (DR) can be implemented, such as intelligent direct load control and price policies [4].

AMI is the deployment of interconnected smart meters that enable two-way communication in order to have continuous and timely monitoring of meter data, outage reporting, and service connect/disconnect. In addition, the AMI network is able to reconfigure due to a failure in communications and to interconnect to utility billing [5], energy market, and pricing policies [4], [6]–[8]. It is envisioned that smart meters will encourage consumers to conserve energy by helping them maintain the quantity and cost of their energy consumption [9].

By the year 2014, worldwide deployment of smart meters is expected to reach about 212 million units. Yet, there are many issues and challenges needing to be resolved before the realization of such visionary network. For instance, the process of replacing the existing energy meters with a smart meter system is an area challenge for utility companies. Secondly, to have the full advantage of the smart meter system, we need all the appliances and devices in the distribution and metering network to be integrated in the communication network. Thirdly, considering the communications aspects, we can envision several issues such as terrestrial difficulties and their impact on the signal quality and availability, the network range, and its impact on the network coverage. All these issues lead to low AMI performance [9]. Therefore, intelligent schemes have to be developed to overcome the communication problems and in particular the channel reliability. Recently, several researchers have explored the impact of channel reliability on the network performance [10]–[13].

Tuite [10] proposed an adaptive forward error correction mechanism for smart grids environments, including 500 kV outdoor substation and underground transformer vaults in order to increase network reliability in wireless sensor network-based smart grid systems. Considering the transmission loss of voltage/phase information and its impact on power generation, this has been investigated in [12]. The power demand estimation and how it is affected by packet loss due to wireless channel impairments has been studied in [14]. This paper focused on optimizing the cost of power supply given demand uncertainty due to packet loss. In this paper, a queuing model is used to quantify the packet loss due to congestion at data aggregation unit (DAU), and then optimize the transmission rate from the DAU to minimize the impact of packet loss.

In the same direction, Zheng *et al.* [13] presented methodologies for deriving reliability performance of wireless

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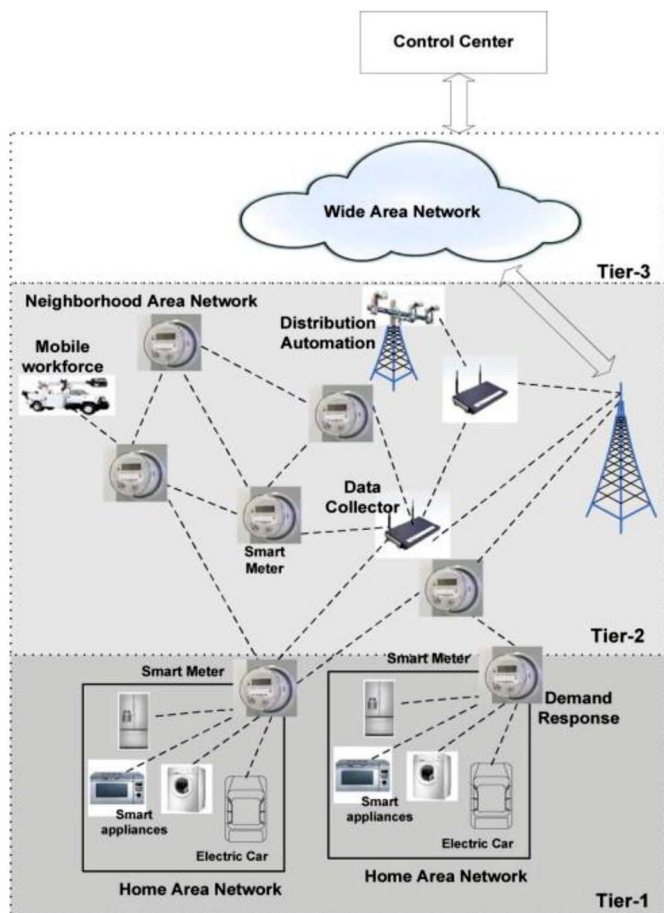


Fig. 1. Multitier smart grid communications network with HANs, NANs, and WANs [16].

communication networks to support DR control. Two wireless network architectures were considered: 1) the single-hop infrastructure-based network; and 2) the multihop mesh network. The communication channel impairments such as multipath fading and shadowing are infused in the channel model and according to the outage probability and packet loss were estimated.

From the above brief survey of the existing work in the literature, we can observe that a little has been done to integrate the channel impairments and the bandwidth availability in one model to obtain a strategic policy for DR, which can guarantee both customers satisfaction and energy providers, while minimize energy fluctuation. Game theory has been proposed as a powerful tool to solve many issues related to smart grids [15].

In this paper, we are exploiting game theory to design a control strategy for the DR from energy retailers for neighborhood-area-network (NAN) customers. NAN comprises a collection of AMI that may represent a number of buildings/houses. Fig. 1 illustrates the multitier structure for the future smart grid. We can observe in Tier-1 that each smart meter is interconnecting (either wirelessly or via Ethernet connection) several smart appliances inside the house. Moreover, Tier-2 (NAN) interconnects the smart meters via a mesh network that facilitates reliable and fast interconnection among many AMIs. Finally, Tier-3 represents a wide range of network technologies (e.g., fiber optics, mobile wireless networks such as 4G)

that connect energy provider/retailers with NAN. It is obvious that exchanging DR messages over these heterogeneous networks may lead to multiple transmission errors.

The rest of this paper is organized as follows. The game theoretic model for retailer-customer problem is described in Section II. In Section III, we describe the main features and conditions of our game model. In Section IV, we present the mathematical programming formulation along with a discussion and some propositions regarding the solution characteristics. Our computational experiments on randomly generated SMGDSM games are described in Section V. Finally, Section VI concludes this paper.

II. GAME THEORETIC MODELS FOR SMART GRIDS

Many authors proposed game theoretic models to solve some issues related to smart grids. Cui *et al.* [17] proposed two models of price determination for energy companies. In their approach, the energy price competition is modeled as a n -person game where one's price strategy affects the payoffs of others. For their simple first model, a Nash equilibrium [18] solution is presented and proven to be unique under some assumptions. Their second model is more sophisticated since it involves factors such as the cost of energy generation and the homeowner's reaction to the change of energy usage as a function of energy price. For this second model, Cui *et al.* [17] presented a practical solution such that no energy company would be able to increase its expected profit by adjusting the price function.

Rajasekharan *et al.* [19] proposed a model for smart grid households equipped with energy storage systems in a local neighborhood. In their model, the users cooperate to trade their real-time energy, supplied by an energy company, and their stored energy in order to minimize their consumptions' cost. The cooperation between users is motivated by the difference in pricing mechanisms adopted by utility companies that serve the locality. Particularly, they focused on a simple two-user two-good exchange economy market to illustrate their approach. They prove that the goods trading market prices regulate themselves in such a way that both users reduce their costs.

Atzeni *et al.* [20], [21] focused on smart grids in which the demand-side involves traditional users as well as users owning some kind of distributed energy sources and/or energy storage devices. The latter users are equipped with independent central units which enable them to regulate their monetary energy expense by producing or storing energy rather than just purchasing their energy needs from the grid. Atzeni *et al.* [20], [21] formulated the smart grid DSM problem as a noncooperative game and analyzed the existence of optimal strategies. To that aim, they presented a distributed algorithm to be run on the users' smart meters.

Nguyen *et al.* [22] considered a smart power system with distributed users requesting dynamically their demands to an energy provider. Simultaneously, the energy provider dynamically updates the energy prices based on the load profiles of the users. In their model, the users try to minimize the peak-to-average ratio (PAR) of the power system by charging

for their batteries at low-demand periods and discharging the energy at high-demand periods. To do so, Nguyen *et al.* [22] proposed a distributed DSM algorithm using a game theoretical approach where each user intends to minimize its total energy cost. Their simulation results showed that the proposed algorithm simultaneously minimizes the PAR and the total energy cost. We now introduce the game theoretic model proposed in this paper.

III. NEW GAME THEORETIC MODEL FOR SMART GRIDS

Our new theoretic model consists in a noncooperative multi-agent game where users and providers (e.g., retailers) interact to regulate a smart grid's demand. Let n be the total number of users (households) and p the total number of providers (energy companies or retailers). We introduce a $(n + p)$ -person noncooperative game where each agent aims at maximizing his own utility depending on a number of conditions. For each user i , the following conditions are considered.

- 1) The total amount of energy requested from all providers cannot exceed his demand.
- 2) The average amount of energy requested from each provider j cannot exceed the amount of energy supplied by j to i .

Simultaneously, for each provider j , the following conditions are considered.

- 1) The total amount of energy supplied to all users cannot exceed their own (production and delivery) capacity.
- 2) The amount of energy supplied by provider j to each user i cannot exceed the average amount of energy requested by i from j .

The use of the expression “average amount of energy requested” models the probability of having a difference between the theoretical amount requested and the real amount requested. The theoretical amount represents the amount of the requests sent to the provider, while the real amount represents the amount of the requests received by the provider. This difference is essentially due to the average packet error rate ρ_{ij} measured experimentally during the transmission of the information from i to j over the wireless network deployed in the NAN.

IV. MATHEMATICAL PROGRAMMING FORMULATION

We define the variables, parameters, and the utility functions used in our formulation in Table I. The variable x_{ij} represents the proportion of energy user i requests from provider j . Similarly, the variable y_{ij} represents the proportion of energy provider j delivers to user i . The main purpose of the game theoretical model we propose is to make the users and the providers interact while all conditions listed above are satisfied and the utility function of each agent is maximized. Formally, a Nash equilibrium [18] is a situation where each agent maximizes his own payoff given what the other agents did. For the smart grid's demand management problem, we define a Nash equilibrium as a situation where users and providers maximize simultaneously their individual utility functions. None of the users or providers would have any interest to change

TABLE I
DEFINITION OF THE VARIABLES, PARAMETERS, AND UTILITY FUNCTIONS

Variable	Type	Description
x_{ij}	Real	proportion i requests from j ;
x_i	Vector	proportions i requests from all providers;
y_{ij}	Real	proportion j delivers to i ;
y_j	Vector	proportions j delivers to all users at;
Parameter		
D_i	Real	demand of i ;
C_j	Real	capacity of j ;
ρ_{ij}	Real	packet error rate between i and j ;
Function		
$f_{ij}(x_{ij}, y_{ij})$	Real	utility for i of the service from j ;
$g_{ij}(x_{ij}, y_{ij})$	Real	utility for j of the service to i .

unilaterally his proportions vector x_i or y_j . The following Definition 1 of a Nash equilibrium for the smart grid's demand management game can be stated.

A. Problem Formulation

For the smart grid's game we propose, a Nash equilibrium is a situation where each agent (user or provider) maximizes his own utility function given the other agents' decisions. Hence, for any user i his requests vector \hat{x}_i should belong to the set of best requests vectors which maximize his utility. Similarly, for any provider j his delivery vector \hat{y}_j should belong to the set of best deliveries vectors which maximize his utility. The following formal definition can be stated.

Definition 1: For the smart grid's game, a Nash equilibrium is a vector of strategies $(\hat{x}_1, \dots, \hat{x}_n, \hat{y}_1, \dots, \hat{y}_p)$, such that for each user i and for each provider j , respectively, we have

$$\hat{x}_i \in \operatorname{argmax}_{x_{ij}} \sum_{j=1}^p f_{ij}(x_{ij}, \hat{y}_j)$$

$$\text{subject to } \sum_{j=1}^p x_{ij} \leq 1 \quad (1)$$

$$(1 - \rho_{ij}) x_{ij} \leq y_{ij}, \forall j \quad (2)$$

$$x_{ij} \geq 0 \quad (3)$$

and

$$\hat{y}_j \in \operatorname{argmax}_{y_{ij}} \sum_{i=1}^n g_{ij}(\hat{x}_i, y_{ij})$$

$$\text{subject to } \sum_{i=1}^n y_{ij} D_i \leq C_j \quad (4)$$

$$y_{ij} \leq (1 - \rho_{ij}) \hat{x}_i, \forall i \quad (5)$$

$$y_{ij} \geq 0. \quad (6)$$

Constraint (1) states that the sum of the proportions requested by the user i cannot exceed one. Constraint (2) states that the average proportion requested by the user i from provider j cannot exceed the proportion delivered by provider j to user i . Constraint (3) states that the proportions x_{ij} are positive. Constraint (4) states that the total quantity delivered by provider j to all users cannot exceed the capacity of the provider j . Constraint (5) states that the proportion delivered by the provider j to user i cannot exceed the average proportion requested by user i from provider j . Finally, constraint (6)

states that the proportions y_{ij} are positive. For simplification, we will suppose in the next development that the utility functions are of quadratic forms. The main motivation behind this assumption is to obtain a linear objective function for each agent involved in the smart grid's game as it is shown in Definition 2.

Assumption 1: The utility function for each user i with respect to each provider j is such that $f_{ij}(x_{ij}, y_{ij}) = x_{ij}q_{ij}y_{ij}$. The utility function for each provider j with respect to each user i is such that $g_{ij}(x_{ij}, y_{ij}) = x_{ij}p_{ij}y_{ij}$. The parameter q_{ij} represents the quality of the service delivered by provider j to user i and the parameter p_{ij} represents the payoff of the service delivered by provider j to user i .

Since each player i or j controls only his decision x_i or y_j , and following the Assumption 1, the definition of a Nash equilibrium for the smart grid's management game can be stated using linear programming.

Definition 2: For the smart grid's game, a Nash equilibrium is a vector of strategies $(\hat{x}_1, \dots, \hat{x}_n, \hat{y}_1, \dots, \hat{y}_p)$, such that for each user i and for each provider j , respectively, we have

$$\begin{aligned} \hat{x}_i &\in \operatorname{argmax}_{x_{ij}} \sum_{j=1}^p x_{ij}q_{ij}\hat{y}_{ij} \\ \text{subject to } &\sum_{j=1}^p x_{ij} \leq 1 \quad (1 - \rho_{ij}) x_{ij} \leq \hat{y}_{ij}, \forall j, x_{ij} \geq 0 \end{aligned}$$

and

$$\begin{aligned} \hat{y}_j &\in \operatorname{argmax}_{y_{ij}} \sum_{i=1}^n \hat{x}_i p_{ij} y_{ij} \\ \text{subject to } &\sum_{i=1}^n y_{ij} D_i \leq C_j y_{ij} \leq (1 - \rho_{ij}) \hat{x}_i, \forall i, y_{ij} \geq 0. \end{aligned}$$

From duality theory, we obtain the following dual programs for each user i and for each provider j , respectively:

$$\begin{aligned} \min_{\alpha_{ij}, \delta_i} &\sum_{j=1}^p \alpha_{ij} \hat{y}_{ij} + \delta_i \\ \text{subject to } &\alpha_{ij} + (1 - \rho_{ij}) \delta_i \geq p_{ij} \hat{y}_{ij}, \forall j \quad (7) \\ &\alpha_{ij}, \delta_i \geq 0 \quad (8) \end{aligned}$$

and

$$\begin{aligned} \min_{\beta_j, \lambda_{ij}} &C_j \beta_j + \sum_{i=1}^n (1 - \rho_{ij}) \hat{x}_i \lambda_{ij} \\ \text{subject to } &D_i \beta_j + \lambda_{ij} \geq \hat{x}_i p_{ij}, \forall i \quad (9) \\ &\beta_j, \lambda_{ij} \geq 0. \quad (10) \end{aligned}$$

Hence, the following definition of a Nash equilibrium for the smart grid's management game can be stated using dual linear programming.

Definition 3: Given a smart grid's game, for each Nash equilibrium $(\hat{x}_1, \dots, \hat{x}_n, \hat{y}_1, \dots, \hat{y}_p)$, there exist for each user i at least one appropriate positive variable $\hat{\delta}_i$ and at least one appropriate vector of positive variables $\hat{\alpha}_i$, and for each provider j there exist at least one appropriate positive variable $\hat{\beta}_j$ and at least one appropriate vector of positive variables $\hat{\lambda}_j$,

such that for each user i and for each provider j , respectively, we have

$$\begin{aligned} (\hat{\alpha}_i, \hat{\delta}_i) &\in \operatorname{argmin}_{\alpha_{ij}, \delta_i} \sum_{j=1}^p \alpha_{ij} \hat{y}_{ij} + \delta_i \\ \text{subject to } &\alpha_{ij} + (1 - \rho_{ij}) \delta_i \geq q_{ij} \hat{y}_{ij}, \forall j, \alpha_{ij}, \delta_i \geq 0 \end{aligned}$$

and

$$\begin{aligned} (\hat{\lambda}_j, \hat{\beta}_j) &\in \operatorname{argmin}_{\beta_j, \lambda_{ij}} C_j \beta_j + \sum_{i=1}^n (1 - \rho_{ij}) \hat{x}_i \lambda_{ij} \\ \text{subject to } &D_i \beta_j + \lambda_{ij} \geq \hat{x}_i p_{ij}, \forall i, \beta_j, \lambda_{ij} \geq 0. \end{aligned}$$

The primal-dual optimality conditions can be expressed as follows. For each user i , and for each provider j , respectively, we have

$$(\hat{\alpha}_{ij} + (1 - \rho_{ij}) \hat{\delta}_i - q_{ij} \hat{y}_{ij}) \hat{x}_{ij} = 0, \forall j \quad (11)$$

$$(D_i \hat{\beta}_j + \hat{\lambda}_{ij} - \hat{x}_{ij} p_{ij}) \hat{y}_{ij} = 0, \forall i. \quad (12)$$

These conditions can be linearized using the binary variables u_{ij} and v_{ij} , and the large real parameter L as shown in [24]

$$\begin{aligned} (\hat{\alpha}_{ij} + (1 - \rho_{ij}) \hat{\delta}_i - q_{ij} \hat{y}_{ij}) \hat{x}_{ij} &= 0, \forall i, j \\ \Rightarrow \begin{cases} \hat{\alpha}_{ij} + (1 - \rho_{ij}) \hat{\delta}_i - q_{ij} \hat{y}_{ij} \leq L u_{ij} & (13) \\ \hat{x}_{ij} + u_{ij} \leq 1 & (14) \\ u_{ij} \text{ binary} & (15) \end{cases} \end{aligned}$$

$$\begin{aligned} (D_i \hat{\beta}_j + \hat{\lambda}_{ij} - \hat{x}_{ij} p_{ij}) \hat{y}_{ij} &= 0, \forall i, j \\ \Rightarrow \begin{cases} D_i \hat{\beta}_j + \hat{\lambda}_{ij} - \hat{x}_{ij} p_{ij} \leq L v_{ij} & (16) \\ \hat{y}_{ij} + v_{ij} \leq 1 & (17) \\ v_{ij} \text{ binary.} & (18) \end{cases} \end{aligned}$$

B. Nash Equilibrium Conditions

The following Proposition 1 formally compiles all the conditions to be satisfied by a Nash equilibrium.

Proposition 1: For the smart grid's demand management game, any Nash equilibrium $(\hat{x}_1, \dots, \hat{x}_n, \hat{y}_1, \dots, \hat{y}_p)$, satisfies the following conditions:

$$\begin{aligned} \sum_{j=1}^p \hat{x}_{ij} &\leq 1, \quad \forall i \\ \sum_{i=1}^n \hat{y}_{ij} D_i &\leq C_j, \quad \forall j \\ (1 - \rho_{ij}) \hat{x}_{ij} &\leq \hat{y}_{ij}, \quad \forall i, j \\ \hat{y}_{ij} &\leq (1 - \rho_{ij}), \quad \forall i, j \\ \hat{\alpha}_{ij} + (1 - \rho_{ij}) \hat{\delta}_i &\geq q_{ij} \hat{y}_{ij}, \quad \forall i, j \\ D_i \hat{\beta}_j + \hat{\lambda}_{ij} &\geq \hat{x}_{ij} p_{ij}, \quad \forall i, j \\ \hat{\alpha}_{ij} + (1 - \rho_{ij}) \hat{\delta}_i - q_{ij} \hat{y}_{ij} &\leq L u_{ij}, \quad \forall i, j \\ D_i \hat{\beta}_j + \hat{\lambda}_{ij} - \hat{x}_{ij} p_{ij} &\leq L v_{ij}, \quad \forall i, j \\ \hat{x}_{ij} + u_{ij} &\leq 1, \quad \forall i, j \\ \hat{y}_{ij} + v_{ij} &\leq 1, \quad \forall i, j \\ \hat{x}_{ij}, \hat{y}_{ij}, \hat{\alpha}_{ij}, \hat{\delta}_i, \hat{\beta}_j, \hat{\lambda}_{ij} &\geq 0 \\ u_{ij}, v_{ij} &\text{ binaries.} \end{aligned}$$

Proof: Any Nash equilibrium satisfies the primal conditions of Definition 1 and the dual conditions of Definition 2 as well as all complementary slackness conditions. Hence, any Nash equilibrium satisfies the conditions of Proposition 1. ■

It is clear that at any Nash equilibrium, we would have $(1 - \rho_{ij})\hat{x}_{ij} = \hat{y}_{ij}$, for each user i and each provider j . In the following development, we propose to compute the Nash equilibrium of the smart grid's DSM game (SMGDSM) which optimizes one of the following objectives:

- 1) maximization of the minimum sum of proportions satisfied for the SMGDSM game users;
- 2) maximization of the minimum utility for the SMGDSM game users;
- 3) maximization of the minimum utility for the SMGDSM game providers.

The master problem (P) corresponds to a 0–1 mixed program resulting from the conditions of Proposition 1

$$\begin{aligned}
 & \max_{x_{ij}, y_{ij}, \alpha_{ij}, \delta_i, \lambda_{ij}, \beta_j} \\
 & \text{subject to } Z \sum_{j=1}^p x_{ij} \leq 1, \quad \forall i \\
 & \sum_{i=1}^n y_{ij} D_i \leq C_j, \quad \forall j \\
 & (1 - \rho_{ij}) x_{ij} \leq y_{ij}, \quad \forall i, j \\
 & y_{ij} \leq (1 - \rho_{ij}), \quad \forall i, j \\
 & \alpha_{ij} + (1 - \rho_{ij}) \delta_i \geq q_{ij} y_{ij}, \quad \forall i, j \\
 & D_i \beta_j + \lambda_{ij} \geq x_{ij} p_{ij}, \quad \forall i, j \\
 & \alpha_{ij} + (1 - \rho_{ij}) \delta_i - q_{ij} y_{ij} \leq L u_{ij}, \quad \forall i, j \\
 & D_i \beta_j + \lambda_{ij} - x_{ij} p_{ij} \leq L v_{ij}, \quad \forall i, j \\
 & x_{ij} + u_{ij} \leq 1, \quad \forall i, j \\
 & y_{ij} + v_{ij} \leq 1, \quad \forall i, j \\
 & x_{ij}, y_{ij}, \alpha_{ij}, \delta_i, \beta_j, \lambda_{ij} \geq 0 \\
 & u_{ij}, v_{ij} \text{ binaries.}
 \end{aligned}$$

Note that the first objective function $Z = Z_1$ chosen intends to generate a Nash equilibrium which maximizes the minimum sum of proportions satisfied of the SMGDSM game user's needs

$$\begin{aligned}
 & \max Z = Z_1 \\
 & \text{subject to } Z_1 \leq \sum_{j=1}^p x_{ij}, \quad \forall i = 1, \dots, n. \quad (19)
 \end{aligned}$$

In the case where $Z = Z_1$ and the conditions (19) are added, the master program (P) becomes a 0–1 mixed linear program. The second possible objective function Z_2 chosen intends to generate a Nash equilibrium which maximizes the minimum utility of the SMGDSM game users. Since the utility of the SMGDSM game users is quadratic, following Assumption 1, and $y_{ij} = (1 - \rho_{ij})x_{ij}$ at any Nash equilibrium, one can write:

$$f_{ij}(x_{ij}, y_{ij}) = x_{ij} q_{ij} y_{ij} = (1 - \rho_{ij}) q_{ij} x_{ij}^2.$$

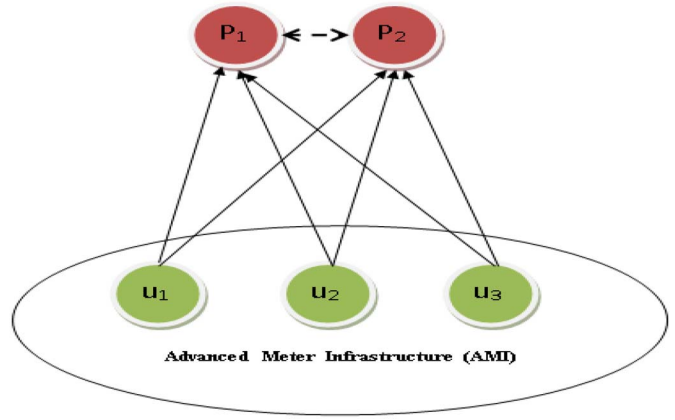


Fig. 2. Example of AMI where a group of users (green nodes) communicates their demands with two providers (red nodes); each provider may exchange information with other providers.

Hence, the second objective function $Z = Z_2$ is such that

$$\begin{aligned}
 & \max Z = Z_2 \\
 & \text{subject to } Z_2 \leq \sum_{j=1}^p (1 - \rho_{ij}) q_{ij} x_{ij}^2, \quad \forall i = 1, \dots, n. \quad (20)
 \end{aligned}$$

In the case where $Z = Z_2$ and the conditions (20) are added, the master program (P) becomes a 0-1 mixed quadratic program. The third possible objective function Z_3 chosen intends to generate a Nash equilibrium which maximizes the minimum utility of the SMGDSM game providers. Since the utility of the SMGDSM game providers is quadratic, following Assumption 1, and $y_{ij} = (1 - \rho_{ij})x_{ij}$ at any Nash equilibrium, one can write:

$$g_{ij}(x_{ij}, y_{ij}) = x_{ij} p_{ij} y_{ij} = (1 - \rho_{ij}) p_{ij} x_{ij}^2.$$

Hence, the second objective function $Z = Z_3$ is such that

$$\begin{aligned}
 & \max Z_3 \\
 & \text{subject to } Z_3 \leq \sum_{i=1}^n (1 - \rho_{ij}) q_{ij} x_{ij}^2, \quad \forall j = 1, \dots, p. \quad (21)
 \end{aligned}$$

In the case where $Z = Z_3$ and the conditions (21) are added, the master program (P) becomes also a 0-1 mixed quadratic program.

C. Discussion

Fig. 2 illustrates a simple schematic connected AMI where three users and two providers are exchanging DR messages.

In this example, users and providers exchange demand, price, and capacity information over wireless channels. Each user node may represent a group of homes connected via one controller. In this example, we assume a noncooperative game scenario. There is no communication among the users regarding their demand, strategy, etc. In addition, the providers can possibly exchange information about their loads.

Let Q and P be the utility matrices of its corresponding smart grid's demand management game. In this example, we use positive and negative utility values. These values are continuously monitored and estimated by individual users based

on the pros and cons of receiving power from this specific provider [7], [9], [25], [26]. For instance, user#1 does not prefer to receive power from provider#2 and hence the announced utility value is “-5,” while it prefers to receive power from provider#1. On the other hand, user#2 has the opposite preference. Later, in Assumption 4.8, we will introduce a more practical approach to define these payoffs

$$Q = \begin{pmatrix} 2 & -5 \\ -3 & 4 \\ 4 & 2 \end{pmatrix} \text{ and } P = \begin{pmatrix} 1 & 4 \\ 3 & 5 \\ -9 & 4 \end{pmatrix}.$$

Let also D and C , respectively, be the users demand and providers capacity vectors

$$D = \begin{pmatrix} 3 \\ 10 \\ 7 \end{pmatrix} \text{ and } C = (10 \quad 10).$$

Considering the maximization of the minimum sum of proportions satisfied Z_1 and¹ $\rho_{ij} = 0.01$ ($\forall i, j$), the Nash equilibrium (X^*, Y^*) is found by the state-of-the-art optimization software Gurobi 5.6 [23], such that

$$X^* = \begin{pmatrix} 1 & 0 \\ 0 & 0.5942 \\ 0 & 0.5942 \end{pmatrix} \text{ and } Y^* = \begin{pmatrix} 0.99 & 0 \\ 0 & 0.5882 \\ 0 & 0.5882 \end{pmatrix}.$$

The nondominated Nash equilibrium found for the previous numerical example suggests the following insights.

Insight 1: Any user would prefer delaying its demand if the only remaining possibility provides a negative payoff, as in the case of user 2 with respect to provider 1, where only 59.42% of his need is satisfied.

Insight 2: Any provider would prefer not responding to any request which generates a negative payoff. A zero quantity is then preferred to any nonzero quantity sent, as in the case of provider 1 with respect to user 3.

Insight 3: The packet error rate is directly affecting the total delivered power by the concerned providers.

The immediate implication of the insights 1 and 2 is that if the providers' total capacity enables the smart grid to satisfy totally all the users demands with nonnegative payoffs for both users and providers, then any user would prefer not delaying its demand.

Insight 4: Any user would prefer not delaying its demand if he can be served with nonnegative payoffs for him and his provider.

D. Solution Characteristics

While the previous insights are straightforward, the following Proposition 2 shows that if all the entries of the payoff matrices Q and P are nonnegative, and if it is possible to satisfy all the users demands with a solution such that the variables x_{ij} are either equal to zero or one, then this solution is a nondominated Nash equilibrium. This issue is important as in practice users may prefer to request energy from a single retailer and not from several ones. In the proof of the Proposition 2, we show that it is always possible in this case to find at least

one feasible solution satisfying all the conditions of a Nash equilibrium.

Proposition 2: If the entries of the payoff matrices Q and P are all nonnegative, let $X^* = (x_{ij}^*)$ be a vector of variables such that all the users demands are totally satisfied and the entries x_{ij}^* are either equal to zero or one, then $X^* = (x_{ij}^*)$ is a nondominated Nash equilibrium for the smart grid's demand management game.

Proof: If $X^* = x_{ij}^*$ is a vector of variables such that all the users demands are totally satisfied and the entries x_{ij}^* are either equal to zero or one. In the first part of the proof, we show that it is always possible to find a feasible setting for the variables δ_i and α_{ij} if the entries of the matrix Q are all positive. In the second part of the proof, we show that it is always possible to find a feasible setting for the variables β_j and λ_{ij} if the entries of the matrix P are all positive. Finally, we conclude that the Nash equilibrium is nondominated.

1) *Part 1:* For a given user i , let j^* be the index of the provider satisfying the whole demand of user i and let j^0 be the index of any other provider. Hence, we have $x_{ij^*} = 1$ and $x_{ij^0} = 0$. Thus, the complementary slackness condition $(\alpha_{ij} + (1 - \rho_{ij})\delta_i - q_{ij}y_{ij})x_{ij} = 0$ is satisfied for each j^0 , since $x_{ij^0} = 0$. For the remaining index j^* , the only possibility to satisfy the complementary slackness condition is to set $\delta_i = \alpha_{ij^*}/(1 - \rho_{ij} - q_{ij^*})$, since $y_{ij^*} = (1 - \rho)x_{ij^*} = 1 - \rho$. Because $q_{ij^*} \geq 0$, one feasible solution would be to set $\alpha_{ij^*} = (1 - \rho)q_{ij^*}$. This way $\delta_i = 0$ and by substituting in the condition $(\alpha_{ij^0} + (1 - \rho)\delta_i) \geq q_{ij^0}y_{ij^0}$, with $y_{ij^0} = 0$, any solution where $\alpha_{ij^0} \geq 0$ is feasible.

2) *Part 2:* For a given provider j , let i^* be the index of any user i for which the demand is totally satisfied by j . Let also i^0 be the index of any other user. Hence, $y_{i^*j} = 1$ and $y_{i^0j} = 0$. Thus, the complementary slackness condition $(D_i\beta_j + \lambda_{ij} - x_{ij}p_{ij})y_{ij}$ is satisfied for each i^0 , since $y_{i^0j} = 0$. For the remaining indices i^* , the only possibility to satisfy the complementary slackness condition is to set $\beta_j = p_{i^*j} - \lambda_{i^*j}/D_{i^*}$, since $y_{i^*j} = (1 - \rho)x_{i^*j} = 1 - \rho$. Because $p_{i^*j} \geq 0$, one feasible solution would be to set $\lambda_{i^*j} = p_{i^*j}$. This way $\beta_j = 0$, and by substituting in the condition $D_{i^0}\beta_j + \lambda_{i^0j} \geq x_{i^0j}p_{i^0j}$, with $x_{i^0j} = 0$, any solution where $\lambda_{i^0j} \geq 0$ is feasible.

3) *Conclusion:* Since $X^* = (x_{ij}^*)$ is a vector of variables such that all the users' demands are totally satisfied, then $X^* = (x_{ij}^*)$ defines a nondominated Nash equilibrium. A feasible setting of the variables would be

$$y_{ij}^* = (1 - \rho)x_{ij}^*$$

$$\delta_i = 0$$

$$\alpha_{ij^*} = (1 - \rho)q_{ij^*}$$

$$\alpha_{ij^0} = 0$$

$$\beta_j = 0$$

$$\lambda_{i^*j} = p_{i^*j}$$

$$\lambda_{i^0j} = 0. \quad \blacksquare$$

Based on Proposition 2, we can set the entries of the payoff matrices P and Q to reasonable values using the user's demand and the provider's capacity.

¹The average value of the packet error rate between each pair user-provider.

TABLE II
COMPUTATIONAL RESULTS ON RANDOM
SMGDSM GAMES (SET 1)

Instance	Size		Opt Z^{bin}			
	n	p	Z_1	Z_2	Z_3	Time
Smg01	60	4	1	0.99	611.10*	1167.49*
Smg02	60	4	1	0.98	902.59	76.40
Smg03	60	4	1	0.97	650.68*	855.00*
Smg04	60	4	1	0.99	706.94	804.85
Smg05	60	4	1	3.96	376.71*	1116.7*
Smg06	60	4	1	3.76	389.70	871.52
Smg07	60	4	1	2.97	481.80	500.97
Smg08	60	4	1	1.94	494.60	186.15
Smg09	60	4	1	1.92	485.32	156.35
Smg10	60	4	1	0.92	679.09*	1513.43*
Smg11	60	3	1	0.99	2555.52	85.09
Smg12	60	3	1	1.92	1664.46	10.54
Smg13	60	3	1	1.88	2043.29	2.25
Smg14	60	3	1	2.82	1702.50	4.64
Smg15	60	3	1	0.91	2284.69	5.99
Smg16	60	3	1	1.84	1887.75	70.07
Smg17	60	3	1	0.98	2911.65	44.83
Smg18	60	3	1	2.79	1758.79	10.85
Smg19	60	3	1	0.96	2592.55	9.14
Smg20	60	3	1	1.00	2560.78	3.27
Smg21	60	2	1	2.00	7348.45	0.62
Smg22	60	2	1	0.90	7919.92	0.53
Smg23	60	2	1	0.99	9016.48	0.35
Smg24	60	2	1	0.99	7943.13	0.42
Smg25	60	2	1	0.87	7858.67	0.16
Smg26	60	2	1	0.96	9377.77	0.31
Smg27	60	2	1	0.95	8953.61	0.62
Smg28	60	2	1	2.91	7211.32	0.16
Smg29	60	2	1	1.92	7978.35	0.49
Smg30	60	2	1	0.99	9672.26	0.37
Average	90	3	1	1.63	3700.68	249.99

TABLE III
COMPUTATIONAL RESULTS ON RANDOM
SMGDSM GAMES (SET 2)

Instance	Size		Opt Z^{bin}			
	n	p	Z_1	Z_2	Z_3	Time
Smg31	50	5	0	0	137.64*	2492.12
Smg32	50	5	0	0	410.57*	4776.49*
Smg33	50	5	0	0	146.42*	1735.39*
Smg34	50	5	0	0	172.83	2381.73
Smg35	50	5	0	0	121.77*	1552.44*
Smg36	50	5	0	0	174.89*	1837.89*
Smg37	50	5	0	0	222.71	2926.21
Smg38	50	5	0	0	119.97*	1437.86*
Smg39	50	5	0	0	153.21*	2536.25*
Smg40	50	5	0	0	167.15*	1196.66*
Smg41	75	3	0	0	1150.91	10.50
Smg42	75	3	0	0	1373.12*	1210.59*
Smg43	75	3	0	0	1379.69*	4222.25*
Smg44	75	3	0	0	975.76*	1372.08*
Smg45	75	3	0	0	1748.73*	804.31*
Smg46	75	3	0	0	1472.93*	1094.07*
Smg47	75	3	0	0	1329.22*	1831.65*
Smg48	75	3	0	0	1154.10*	1037.65*
Smg49	75	3	0	0	1842.33*	1311.88*
Smg50	75	3	0	0	1494.69*	1324.14*
Smg51	100	2	0	0	5100.11	1.20
Smg52	100	2	0	0	5944.39	11.19
Smg53	100	2	0	0	6355.67	27.09
Smg54	100	2	0	0	3934.94	2.48
Smg55	100	2	0	0	3804.93	2.93
Smg56	100	2	0	0	5632.69	15.85
Smg57	100	2	0	0	7016.16	1.24
Smg58	100	2	0	0	4801.88	471.86
Smg59	100	2	0	0	3983.91	3.37
Smg60	100	2	0	0	4776.94	4.59
Average	75	3.33	0	0	2236.68	1254.47

Assumption 2: The entries of the payoff matrices P and Q are defined such that

$$q_{ij} = D_i \text{ and } p_{ij} = \frac{C_i}{D_i}.$$

From the user's point of view, this choice of the utility parameters sets an equivalence between the user's utility and his demand. While from the provider's point of view, this choice penalizes the users with large demands or demands exceeding the provider's capacity.

V. COMPUTATIONAL RESULTS

Our computational experiments on randomly generated SMGDSM games with different size are presented in Tables II and III. These experimental results were obtained under Windows 7, on workstations with 2.4 GHz Intel Core i5 processors, and 2.93 GB RAM. The state-of-the-art software Gurobi 5.6 [16] was used for the optimization of the 0–1 mixed linear programs. Our computational results are obtained on two sets of instances. The first set of instances "Set 1" (smg01–smg30) involves a number of users ranging from 60 to 120 and a number of providers ranging from 2 to 4. For this set of 30 different instances, all the user's needs can be fully satisfied by the providers. The results on the first set of instances are presented in Table II. The second set of instances "Set 2" (smg31–smg60) involves a number of users ranging from 50 to 100 and a number of providers ranging from 2 to 5. For this set of 30 different instances, the user's needs cannot

be fully satisfied by the providers. The results on the second set of instances are presented in Table III.

For the entries in Tables II and III, the column "instance" indicates the name of the instance solved. The column "size" indicates the original size of the generated set of SMGDSM game. For each given size, we have randomly generated ten different SMGDSM games. The columns " n " and " p " indicate, respectively, the number of users and the number of providers. The column "Opt Z^{bin} " indicates the value each of the optimal objective functions Z_1 , Z_2 , and Z_3 , when the x_{ij} variables are considered as binary variables so each user i cannot be served by more than one provider. Since for x_{ij} as binary variables we always have $x_{ij}^2 = x_{ij}$, the nonlinearity appearing with the use of Z_2 or Z_3 as objectives is eliminated. Finally, the column "time" indicates the execution time (in seconds) per game when the objective function Z_3 is chosen.

The entries noted with "*" indicate that the value of Table III objective Z_3 is the best found while the optimization was automatically interrupted and the workstation ran out of memory. Although it is not appearing in these tables, for both sets 1 and 2, the optimal solutions with the objectives Z_1 and Z_2 were obtained in less than 0.05 s.

A single user in our model could refer to a group of domestic users within a city or urban area. We can agree that it is more realistic to assign a pool of neighboring houses to a given provider than it is the case if every single domestic user chooses his own provider. In the case where a large number of users have to be considered, a time upper bound could be set such that the current best solution found by the optimizer is

returned and the optimization process is stopped. In addition, heuristic optimization algorithms such as variable neighborhood search or Tabu search can be used to obtain a good solution quality in reasonable time.

VI. CONCLUSION

In this paper, we proposed and studied a new noncooperative multiagent game theoretic model for managing the demands of a group of smart grid users. Using primal-dual optimality conditions, the Nash equilibrium conditions for the proposed model are set. Under an assumption on the form of the utility functions, we developed a 0-1 mixed programming approach to compute nondominated extreme Nash equilibria. To do so, we have used three different objective functions. With the first objective, we intended to maximize the minimum sum of proportions satisfied for the game users. With the second objective, we intended to maximize the minimum utility of the game users. Finally, with the third objective, we intended to maximize the minimum utility of the game providers. We have presented our computational results on randomly generated smart grid demand management games with different size. As a future work, we shall consider the impact of instantaneous error on heterogeneous users.

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