Basic Business Statistics
11th Edition

Chapter 12

Chi-Square Tests and Nonparametric Tests

Learning Objectives

In this chapter, you learn:

- How and when to use the chi-square test for contingency tables
- How to use the Marascuilo procedure for determining pairwise differences when evaluating more than two proportions
- How and when to use the McNemar test
- How to use the chi-square test for a variance or standard deviation
- How and when to use nonparametric tests

Contingency Tables

Contingency Tables

- Useful in situations involving multiple population proportions
- Used to classify sample observations according to two or more characteristics
- Also called a cross-classification table.

Contingency Table Example

Left-Handed vs. Gender

Dominant Hand: Left vs. Right

Gender: Male vs. Female

- 2 categories for each variable, so called a 2 x 2 table
- Suppose we examine a sample of 300 children

Contingency Table Example

(continued)

Sample results organized in a contingency table:

sample size = n = 300:

120 Females, 12 were left handed

180 Males, 24 were left handed

Gender	Hand Preference: Left	Hand Preference: Right	
Female	12	108	120
Male	24	156	180
	36	264	300

χ^2 Test for the Difference Between Two Proportions

H_0: $\pi_1 = \pi_2$ (Proportion of females who are left handed is equal to the proportion of males who are left handed)

H_1: $\pi_1 \neq \pi_2$ (The two proportions are not the same – hand preference is not independent of gender)

- If H_0 is true, then the proportion of left-handed females should be the same as the proportion of left-handed males
- The two proportions above should be the same as the proportion of left-handed people overall

The Chi-Square Test Statistic

The Chi-square test statistic is:

$$\chi^2_{STAT} = \sum_{all\ cells} \frac{(f_o - f_e)^2}{f_e}$$

- where:
  f_o = observed frequency in a particular cell
  f_e = expected frequency in a particular cell if H_0 is true

χ^2_{STAT} **for the 2 x 2 case has 1 degree of freedom**

(Assumed: each cell in the contingency table has expected frequency of at least 5)

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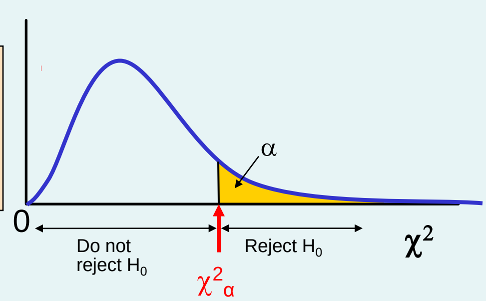
Decision Rule

The χ^2_{STAT} test statistic approximately follows a chi-squared distribution with one degree of freedom

Decision Rule:
If $\chi^2_{STAT} > \chi^2_{\alpha}$, reject H_0, otherwise, do not reject H_0



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Computing the Average Proportion

The average proportion is: $\bar{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{X}{n}$

120 Females, 12 were left handed

180 Males, 24 were left handed

Here:

$$\bar{p} = \frac{12 + 24}{120 + 180} = \frac{36}{300} = 0.12$$

i.e., of all the children the proportion of left handers is 0.12, that is, 12%

Finding Expected Frequencies

- To obtain the expected frequency for left handed females, multiply the average proportion left handed ($\bar{p}$) by the total number of females
- To obtain the expected frequency for left handed males, multiply the average proportion left handed ($\bar{p}$) by the total number of males

If the two proportions are equal, then

P(Left Handed | Female) = P(Left Handed | Male) = .12

i.e., we would expect (.12)(120) = 14.4 females to be left handed
(.12)(180) = 21.6 males to be left handed

Observed vs. Expected Frequencies

Gender	Hand Preference: Left	Hand Preference: Right	
Female	Observed = 12 Expected = 14.4	Observed = 108 Expected = 105.6	120
Male	Observed = 24 Expected = 21.6	Observed = 156 Expected = 158.4	180
	36	264	300

The Chi-Square Test Statistic

Gender	Hand Preference: Left	Hand Preference: Right	
Female	Observed = 12 Expected = 14.4	Observed = 108 Expected = 105.6	120
Male	Observed = 24 Expected = 21.6	Observed = 156 Expected = 158.4	180
	36	264	300

The test statistic is:

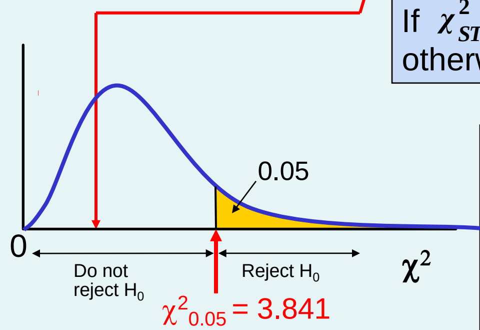
$$\chi^2_{STAT} = \sum_{\text{all cells}} \frac{(f_o - f_e)^2}{f_e}$$

$$= \frac{(12-14.4)^2}{14.4} + \frac{(108-105.6)^2}{105.6} + \frac{(24-21.6)^2}{21.6} + \frac{(156-158.4)^2}{158.4} = 0.7576$$

Decision Rule

The test statistic is $\chi^2_{STAT} = 0.7576$; $\chi^2_{0.05}$ with 1 d.f. = 3.841

Decision Rule:
If $\chi^2_{STAT} > 3.841$, reject H_0, otherwise, do not reject H_0



Here, $\chi^2_{STAT} = 0.7576 < \chi^2_{0.05} = 3.841$, so we do not reject H_0 and conclude that there is not sufficient evidence that the two proportions are different at $\alpha = 0.05$

χ^2 Test for Differences Among More Than Two Proportions

- Extend the χ^2 test to the case with more than two independent populations:

H_0: $\pi_1 = \pi_2 = \cdots = \pi_c$

H_1: Not all of the π_j are equal (j = 1, 2, ⋯, c)

The Chi-Square Test Statistic

The Chi-square test statistic is:

$$\chi^2_{STAT} = \sum_{all\ cells} \frac{(f_o - f_e)^2}{f_e}$$

- Where:
  f_o = observed frequency in a particular cell of the 2 x c table
  f_e = expected frequency in a particular cell if H_0 is true

χ^2_{STAT} for the 2 x c case has (2 - 1)(c - 1) = c - 1 degrees of freedom

(Assumed: each cell in the contingency table has expected frequency of at least 1)

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Computing the Overall Proportion

The overall proportion is:

$$\bar{p} = \frac{X_1 + X_2 + \cdots + X_c}{n_1 + n_2 + \cdots + n_c} = \frac{X}{n}$$

- Expected cell frequencies for the c categories are calculated as in the 2 x 2 case, and the decision rule is the same:

Decision Rule:
If $\chi^2_{STAT} > \chi^2_\alpha$, reject H_0, otherwise, do not reject H_0

Where χ^2_α is from the chi-squared distribution with c – 1 degrees of freedom

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The Marascuilo Procedure

- Used when the null hypothesis of equal proportions is rejected
- Enables you to make comparisons between all pairs
- Start with the observed differences, $p_j - p_{j'}$, for all pairs (for $j \neq j'$) . . .
- . . .then compare the absolute difference to a calculated critical range

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The Marascuilo Procedure

(continued)

- Critical Range for the Marascuilo Procedure:

$$\text{Critical range} = \sqrt{\chi^2_\alpha}\sqrt{\frac{p_j(1-p_j)}{n_j} + \frac{p_{j'}(1-p_{j'})}{n_{j'}}}$$

  - (Note: the critical range is different for each pairwise comparison)
- A particular pair of proportions is significantly different if

$$|p_j - p_{j'}| > \text{critical range for } j \text{ and } j'$$

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Marascuilo Procedure Example

A University is thinking of switching to a trimester academic calendar. A random sample of 100 administrators, 50 students, and 50 faculty members were surveyed

Opinion	Administrators	Students	Faculty
Favor	63	20	37
Oppose	37	30	13
Totals	100	50	50

Using a 1% level of significance, which groups have a different attitude?

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Chi-Square Test Results

H_0: $\pi_1 = \pi_2 = \pi_3$

H_1: Not all of the π_j are equal (j = 1, 2, 3)

Chi-Square Test: Administrators, Students, Faculty

	Admin	Students	Faculty	Total
Favor	63	20	37	120
	60	30	30	
Oppose	37	30	13	80
	40	20	20	
Total	100	50	50	200

Expected (rows 60, 30, 30 and 40, 20, 20)

Observed (rows 63, 20, 37 and 37, 30, 13)

$$\chi^2_{STAT} = 12.792 > \chi^2_{0.01} = 9.2103 \text{ so reject } H_0$$

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Marascuilo Procedure: Solution

Excel Output:

compare

Marascuilo Procedure

Group	Sample Proportion	Sample Size	Comparison	Absolute Difference	Std. Error of Difference	Critical Range	Results
1	0.63	100	1 to 2	0.23	0.084445249	0.2563	Means are not different
2	0.4	50	1 to 3	0.11	0.078606615	0.2386	Means are not different
3	0.74	50	2 to 3	0.34	0.092994624	0.2822	Means are different

At 1% level of significance, there is evidence of a difference in attitude between students and faculty

χ^2 Test of Independence

- Similar to the χ^2 test for equality of more than two proportions, but extends the concept to contingency tables with r rows and c columns

H_0: The two categorical variables are independent
(i.e., there is no relationship between them)

H_1: The two categorical variables are dependent
(i.e., there is a relationship between them)

χ^2 Test of Independence

(continued)

The Chi-square test statistic is:

$$\chi^2_{STAT} = \sum_{all\ \text{cells}} \frac{(f_o - f_e)^2}{f_e}$$

- where:

  f_o = observed frequency in a particular cell of the r x c table

  f_e = expected frequency in a particular cell if H_0 is true

χ^2_{STAT} for the r x c case has (r-1)(c-1) degrees of freedom

(Assumed: each cell in the contingency table has expected frequency of at least 1)

Expected Cell Frequencies

- Expected cell frequencies:

$$f_e = \frac{\text{row total} \times \text{column total}}{n}$$

Where:

row total = sum of all frequencies in the row

column total = sum of all frequencies in the column

n = overall sample size

Decision Rule

- The decision rule is

If $\chi^2_{STAT} > \chi^2_{\alpha}$, reject H_0,
otherwise, do not reject H_0

Where χ^2_{α} is from the chi-squared distribution with $(r-1)(c-1)$ degrees of freedom

Example

- The meal plan selected by 200 students is shown below:

Class Standing	Number of meals per week			Total
	20/week	10/week	none	
Fresh.	24	32	14	70
Soph.	22	26	12	60
Junior	10	14	6	30
Senior	14	16	10	40
Total	70	88	42	200

Example

(continued)

- The hypothesis to be tested is:

H_0: Meal plan and class standing are independent
(i.e., there is no relationship between them)
H_1: Meal plan and class standing are dependent
(i.e., there is a relationship between them)

Example: Expected Cell Frequencies

(continued)

Observed:

Class Standing	Number of meals per week			Total
	20/wk	10/wk	none	
Fresh.	24	32	14	70
Soph.	22	26	12	60
Junior	10	14	6	30
Senior	14	16	10	40
Total	70	88	42	200

Expected cell frequencies if H_0 is true:

Class Standing	Number of meals per week			Total
	20/wk	10/wk	none	
Fresh.	24.5	30.8	14.7	70
Soph.	21.0	26.4	12.6	60
Junior	10.5	13.2	6.3	30
Senior	14.0	17.6	8.4	40
Total	70	88	42	200

Example for one cell:

$$f_e = \frac{\text{row total} \times \text{column total}}{n} = \frac{30 \times 70}{200} = 10.5$$

Example: The Test Statistic

(continued)

- The test statistic value is:

$$\chi^2_{STAT} = \sum_{all\ cells} \frac{(f_o - f_e)^2}{f_e}$$
$$= \frac{(24-24.5)^2}{24.5} + \frac{(32-30.8)^2}{30.8} + \cdots + \frac{(10-8.4)^2}{8.4} = 0.709$$

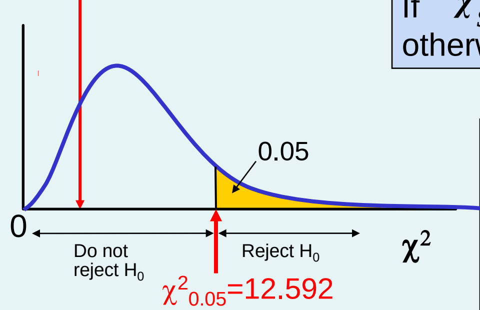
$\chi^2_{0.05}$ = 12.592 from the chi-squared distribution with (4 – 1)(3 – 1) = 6 degrees of freedom

Example: Decision and Interpretation

(continued)

The test statistic is $\chi^2_{STAT} = 0.709$; $\chi^2_{0.05}$ with 6 d.f. = 12.592

Decision Rule:
If χ^2_{STAT} > 12.592, reject H_0, otherwise, do not reject H_0



Here, χ^2_{STAT} = 0.709 < $\chi^2_{0.05}$ = 12.592, so do not reject H_0

Conclusion: there is not sufficient evidence that meal plan and class standing are related at α = 0.05

McNemar Test (Related Samples)

- Used to determine if there is a difference between proportions of two related samples
- Uses a test statistic the follows the normal distribution

McNemar Test (Related Samples)

(continued)

- Consider a 2 X 2 contingency table:

	Condition 2		
Condition 1	Yes	No	Totals
Yes	A	B	A+B
No	C	D	C+D
Totals	A+C	B+D	n

McNemar Test (Related Samples)

(continued)

- The sample proportions of interest are

$$p_1 = \frac{A+B}{n} = \text{proportion of respondents who answer yes to condition 1}$$

$$p_2 = \frac{A+C}{n} = \text{proportion of respondents who answer yes to condition 2}$$

- Test $H_0: \pi_1 = \pi_2$
  (the two population proportions are equal)
  $H_1: \pi_1 \neq \pi_2$
  (the two population proportions are not equal)

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McNemar Test (Related Samples)

(continued)

- The test statistic for the McNemar test:

$$Z_{STAT} = \frac{B - C}{\sqrt{B + C}}$$

where the test statistic Z is approximately normally distributed

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McNemar Test Example

- Suppose you survey 300 homeowners and ask them if they are interested in refinancing their home. In an effort to generate business, a mortgage company improved their loan terms and reduced closing costs. The same homeowners were again surveyed. Determine if change in loan terms was effective in generating business for the mortgage company. The data are summarized as follows:

McNemar Test Example

Survey response before change	Survey response after change		
	Yes	No	Totals
Yes	118	2	120
No	22	158	180
Totals	140	160	300

Test the hypothesis (at the 0.05 level of significance):

H_0: $\pi_1 \geq \pi_2$: The change in loan terms was ineffective

H_1: $\pi_1 < \pi_2$: The change in loan terms increased business

McNemar Test Example

Survey response before change	Survey response after change		
	Yes	No	Totals
Yes	118	2	120
No	22	158	180
Totals	140	160	300

The critical value (0.05 significance) is $Z_{0.05} = -1.645$

The test statistic is:

$$Z_{STAT} = \frac{B - C}{\sqrt{B + C}} = \frac{2 - 22}{\sqrt{2 + 22}} = -4.08$$

Since $Z_{STAT} = -4.08 < -1.645$, you reject H_0 and conclude that the change in loan terms significantly increase business for the mortgage company.

Chi-Square Test for a Variance or Standard Deviation

- A χ^2 test statistic is used to test whether or not the population variance or standard deviation is equal to a specified value:

$$\chi^2_{STAT} = \frac{(n-1)S^2}{\sigma^2}$$

Where n = sample size

S^2 = sample variance

σ^2 = hypothesized population variance

χ^2_{STAT} follows a chi-square distribution with n – 1 d.f.

Wilcoxon Rank-Sum Test for Differences in 2 Medians

- Test two independent population medians
- Populations need not be normally distributed
- Distribution free procedure
- Used when only rank data are available
- Must use normal approximation if either of the sample sizes is larger than 10

Wilcoxon Rank-Sum Test: Small Samples

- Can use when both n_1, n_2 ≤ 10
- Assign ranks to the combined $n_1 + n_2$ sample observations
  - If unequal sample sizes, let n_1 refer to smaller-sized sample
  - Smallest value rank = 1, largest value rank = $n_1 + n_2$
  - Assign average rank for ties
- Sum the ranks for each sample: T_1 and T_2
- Obtain test statistic, T_1 (from smaller sample)

Checking the Rankings

- The sum of the rankings must satisfy the formula below
- Can use this to verify the sums T_1 and T_2

$$T_1 + T_2 = \frac{n(n+1)}{2}$$

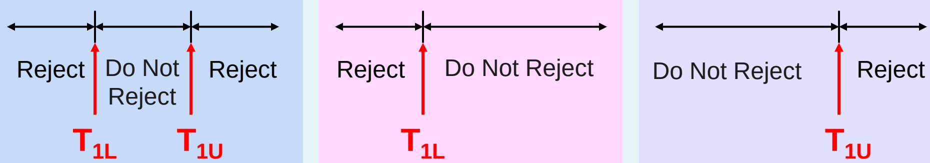
where $n = n_1 + n_2$

Wilcoxon Rank-Sum Test: Hypothesis and Decision Rule

M_1 = median of population 1; M_2 = median of population 2

Test statistic = T_1 (Sum of ranks from smaller sample)

Two-Tail Test	Left-Tail Test	Right-Tail Test
$H_0: M_1 = M_2$	$H_0: M_1 \geq M_2$	$H_0: M_1 \leq M_2$
$H_1: M_1 \neq M_2$	$H_1: M_1 < M_2$	$H_1: M_1 > M_2$



Two-Tail Test	Left-Tail Test	Right-Tail Test
Reject H_0 if $T_1 \leq T_{1L}$ or if $T_1 \geq T_{1U}$	Reject H_0 if $T_1 \leq T_{1L}$	Reject H_0 if $T_1 \geq T_{1U}$

Wilcoxon Rank-Sum Test: Small Sample Example

Sample data are collected on the capacity rates (% of capacity) for two factories.

Are the median operating rates for two factories the same?

- For factory A, the rates are 71, 82, 77, 94, 88
- For factory B, the rates are 85, 82, 92, 97

Test for equality of the population medians at the 0.05 significance level

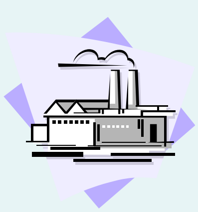
Wilcoxon Rank-Sum Test: Small Sample Example

(continued)

Ranked Capacity values:

Capacity		Rank	
Factory A	**Factory B**	**Factory A**	**Factory B**
71		1	
77		2	
82		3.5	
	82		3.5
	85		5
88		6	
	92		7
94		8	
	97		9
	Rank Sums:	**20.5**	**24.5**

Tie in 3rd and 4th places



Wilcoxon Rank-Sum Test: Small Sample Example

(continued)

Factory B has the smaller sample size, so the test statistic is the sum of the Factory B ranks:

$T_1 = 24.5$

The sample sizes are:

$n_1 = 4$ (factory B)

$n_2 = 5$ (factory A)

The level of significance is $\alpha = .05$

Wilcoxon Rank-Sum Test: Small Sample Example

(continued)

- Lower and Upper Critical Values for T_1 from Appendix table E.8:

n_2	α		n_1	
	One-Tailed	Two-Tailed	4	5
4				
5	.05	.10	12, 28	19, 36
	.025	.05	11, 29	17, 38
	.01	.02	10, 30	16, 39
	.005	.01	--, --	15, 40
6				

$T_{1L} = 11$ and $T_{1U} = 29$

Wilcoxon Rank-Sum Test: Small Sample Solution

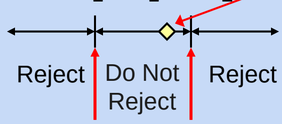
(continued)

- $\alpha = .05$
- $n_1 = 4$, $n_2 = 5$

Two-Tail Test

H_0: $M_1 = M_2$

H_1: $M_1 \neq M_2$



T_{1L}=11 T_{1U}=29

Reject H_0 if $T_1 \leq T_{1L}$=11

or if $T_1 \geq T_{1U}$=29

Test Statistic (Sum of ranks from smaller sample):

$T_1 = 24.5$

Decision:
Do not reject at $\alpha = 0.05$

Conclusion:
There is not enough evidence to prove that the medians are not equal.

Wilcoxon Rank-Sum Test (Large Sample)

- For large samples, the test statistic T_1 is approximately normal with mean μ_{T_1} and standard deviation σ_{T_1} :

$$\mu_{T_1} = \frac{n_1(n+1)}{2}$$

$$\sigma_{T_1} = \sqrt{\frac{n_1 n_2 (n+1)}{12}}$$

- Must use the normal approximation if either n_1 or $n_2 > 10$
- Assign n_1 to be the smaller of the two sample sizes
- Can use the normal approximation for small samples

Wilcoxon Rank-Sum Test (Large Sample)

(continued)

- The Z test statistic is

$$Z_{STAT} = \frac{T_1 - \mu_{T_1}}{\sigma_{T_1}} = \frac{T_1 - \frac{n_1(n+1)}{2}}{\sqrt{\frac{n_1 n_2 (n+1)}{12}}}$$

- Where Z_{STAT} approximately follows a standardized normal distribution

Wilcoxon Rank-Sum Test: Normal Approximation Example

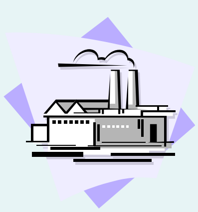
Use the setting of the prior example:

The sample sizes were:

$n_1 = 4$ (factory B)

$n_2 = 5$ (factory A)

The level of significance was $\alpha = .05$



The test statistic was $T_1 = 24.5$

Wilcoxon Rank-Sum Test: Normal Approximation Example

(continued)

$$\mu_{T_1} = \frac{n_1(n+1)}{2} = \frac{4(9+1)}{2} = 20$$

$$\sigma_{T_1} = \sqrt{\frac{n_1 n_2 (n+1)}{12}} = \sqrt{\frac{4(5)(9+1)}{12}} = 4.082$$

- The test statistic is

$$Z_{STAT} = \frac{T_1 - \mu_{T_1}}{\sigma_{T_1}} = \frac{24.5 - 20}{4.0882} = 1.10$$

- Z = 1.10 is not greater than the critical Z value of 1.96 (for α = .05) so we do not reject H_0 – there is not sufficient evidence that the medians are not equal

Wilcoxon Signed Ranks Test

- A nonparametric test for two related populations
- Steps:
  1. For each of n sample items, compute the difference, D_i, between two measurements
  2. Ignore + and – signs and find the absolute values, $|D_i|$
  3. Omit zero differences, so sample size is n′
  4. Assign ranks R_i from 1 to n′ (give average rank to ties)
  5. Reassign + and – signs to the ranks R_i
  6. Compute the Wilcoxon test statistic W as the sum of the positive ranks

Wilcoxon Signed Ranks Test Statistic

- The Wilcoxon signed ranks test statistic is the sum of the positive ranks:

$$W = \sum_{i=1}^{n'} R_i^{(+)}$$

- For small samples ($n' < 20$), use Table E.9 for the critical value of W

Wilcoxon Signed Ranks Test Statistic

- For samples of $n' > 20$, W is approximately normally distributed with

$$\mu_W = \frac{n'(n'+1)}{4}$$

$$\sigma_W = \sqrt{\frac{n'(n'+1)(2n'+1)}{24}}$$

Wilcoxon Signed Ranks Test

- The large sample Wilcoxon signed ranks Z test statistic is

$$Z_{STAT} = \frac{W - \frac{n'(n'+1)}{4}}{\sqrt{\frac{n'(n'+1)(2n'+1)}{24}}}$$

- To test for no median difference in the paired values:

H_0: $M_D = 0$
H_1: $M_D \neq 0$

Kruskal-Wallis Rank Test

- Tests the equality of more than 2 population medians
- Use when the normality assumption for one-way ANOVA is violated
- Assumptions:
  - The samples are random and independent
  - Variables have a continuous distribution
  - The data can be ranked
  - Populations have the same variability
  - Populations have the same shape

Kruskal-Wallis Test Procedure

- Obtain rankings for each value
  - In event of tie, each of the tied values gets the average rank
- Sum the rankings for data from each of the c groups
  - Compute the H test statistic

Kruskal-Wallis Test Procedure

(continued)

- The Kruskal-Wallis H-test statistic:
  (with c – 1 degrees of freedom)

$$H = \left[\frac{12}{n(n+1)} \sum_{j=1}^{c} \frac{T_j^2}{n_j} \right] - 3(n+1)$$

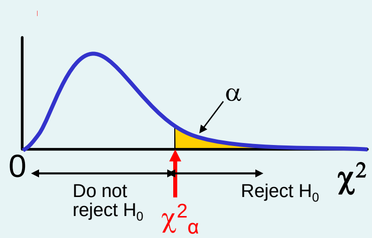
where:

n = sum of sample sizes in all groups
c = Number of groups
T_j = Sum of ranks in the j^{th} group
n_j = Number of values in the j^{th} group (j = 1, 2, … , c)

Kruskal-Wallis Test Procedure

(continued)

- Complete the test by comparing the calculated H value to a critical χ^2 value from the chi-square distribution with c – 1 degrees of freedom



- Decision rule
  - Reject H_0 if test statistic $H > \chi^2_\alpha$
  - Otherwise do not reject H_0

Kruskal-Wallis Example

- Do different departments have different class sizes?

Class size (Math, M)	Class size (English, E)	Class size (Biology, B)
23	55	30
45	60	40
54	72	18
78	45	34
66	70	44

Kruskal-Wallis Example

(continued)

- Do different departments have different class sizes?

Class size (Math, M)	Ranking	Class size (English, E)	Ranking	Class size (Biology, B)	Ranking
23	2	55	10	30	3
41	6	60	11	40	5
54	9	72	14	18	1
78	15	45	8	34	4
66	12	70	13	44	7
	Σ = 44		Σ = 56		Σ = 20

 Chap 12-61

Kruskal-Wallis Example

(continued)

$$H_0: \text{Median}_M = \text{Median}_E = \text{Median}_B$$
$$H_1: \text{Not all population Medians are equal}$$

- The H statistic is

$$H = \left[\frac{12}{n(n+1)} \sum_{j=1}^{c} \frac{T_j^2}{n_j}\right] - 3(n+1)$$
$$= \left[\frac{12}{15(15+1)}\left(\frac{45.5^2}{5} + \frac{55.5^2}{5} + \frac{19^2}{5}\right)\right] - 3(15+1) = 7.12$$

 Chap 12-62

Kruskal-Wallis Example

(continued)

- Compare H = 7.12 to the critical value from the chi-square distribution for 3 – 1 = 2 degrees of freedom and α = 0.05:

$$\chi^2_{0.05} = 5.991$$

Since $H = 7.12 > \chi^2_{0.05} = 5.991$,
reject H_0

There is sufficient evidence to reject that the population medians are all equal

Friedman Rank Test

- Use the Friedman rank test to determine whether c groups (i.e., treatment levels) have been selected from populations having equal medians

H_0: $M_{.1} = M_{.2} = \cdots = M_{.c}$

H_1: Not all $M_{.j}$ are equal (j = 1, 2, …, c)

Friedman Rank Test

(continued)

- Friedman rank test for differences among c medians:

$$F_R = \frac{12}{rc(c+1)} \sum_{j=1}^{c} R_{.j}^2 - 3r(c+1)$$

where $R_{.j}^2$ = the square of the total ranks for group j

r = the number of blocks

c = the number of groups

Friedman Rank Test

(continued)

- The Friedman rank test statistic is approximated by a chi-square distribution with c – 1 d.f.

Reject H_0 if $F_R > \chi_\alpha^2$

Otherwise do not reject H_0

Chapter Summary

- Developed and applied the χ^2 test for the difference between two proportions
- Developed and applied the χ^2 test for differences in more than two proportions
- Applied the Marascuilo procedure for comparing all pairs of proportions after rejecting a χ^2 test
- Examined the χ^2 test for independence
- Applied the McNemar test for proportions from two related samples
- Presented the χ^2 test for a variance or a standard deviation

Chapter Summary

(continued)

- Used the Wilcoxon rank sum test for two population medians
- Presented the Wilcoxon signed ranks test for comparing paired samples
- Applied the Kruskal-Wallis H-test for multiple population medians
- Applied the Friedman rank test for comparing multiple population medians in a randomized block design

