

Q1:

Given the following null and alternative hypotheses:

$$H_0 : \mu_2 \geq \mu_1 \quad v.s \quad H_0 : \mu_2 < \mu_1$$

And the following sample information:

$$\text{Sample I: } n_1 = 33, S_1 = 18, \bar{x}_1 = 43$$

$$\text{Sample II: } n_2 = 30, S_1 = 24, \bar{x}_1 = 40$$

- Develop the appropriate decision rule, assuming a significant level of 0.05 to be used.
- Test the null hypothesis and indicate whether the sample information leads to reject or fail to reject the null hypothesis. Use the test statistic approach.

The hypotheses are: $H_0 : \mu_1 - \mu_2 \leq 0 \quad v.s \quad H_0 : \mu_1 - \mu_2 > 0$

a) Rejection Region:

$n_1 > 30$ and $n_2 = 30$, we can use the z-approximation

Reject H_0 if $Z_{cal} > 1.645$

The test statistic:

$$Z_{cal} = \frac{43 - 40}{\sqrt{\frac{18^2}{33} + \frac{24^2}{30}}} = 0.556911$$

Conclusion:

Do not Reject the null hypothesis.

Q2: The general manager of a chain of fast food chicken restaurants wants to determine how effective their promotional campaigns are. In these campaigns “20% off” coupons are widely distributed. These coupons are only valid for one week. To examine their effectiveness, the executive records the daily gross sales (in \$1,000s) in one restaurant during the campaign and during the week after the campaign ends. The data is shown below. Can they infer at the 5% significance level that sales increase during the campaign?

Day	Sales During Campaign	Sales After Campaign
Sunday	18.1	16.6
Monday	10.0	8.8
Tuesday	9.1	8.6
Wednesday	8.4	8.3
Thursday	10.8	10.1
Friday	13.1	12.3
Saturday	20.8	18.9

ANSWER:

The hypotheses are: $H_0 : \mu_D = 0$

$$H_1 : \mu_D > 0$$

Rejection Region:

$$t > 1.94318$$

The test statistic:

$$\bar{d} = 0.957142857$$

$$S_d = 0.616055038$$

$$t = 4.111$$

Conclusion:

Reject the null hypothesis. Yes

Q3: Two catalysts are being analyzed to determine how they affect the mean yield of a chemical process. Specifically, catalyst 1 is currently in use, but catalyst 2 is acceptable. Since catalyst 2 is cheaper, it should be adopted, providing it does not change the process yield. A test is run in the pilot plant and results in the data shown in Table

Observation number	Catalyst 1	Catalyst 2
1	91.5	89.19
2	94.18	90.95
3	92.18	90.46
4	95.39	93.21
5	91.79	97.19
6	89.07	97.04
7	94.72	91.07

Is there any difference between the mean yields? Assuming a significant level of 0.05 to be used.

	Catalyst 1	Catalyst 2
n	7	7
Sample mean	92.69	92.73
Sample variance	2.206868	3.222489

$S_p = 2.761766$

Assumptions:

1. Normality
2. Independent populations
3. Equal variances (unknown)

The hypotheses are: $H_0 : \mu_1 - \mu_2 = 0$ v.s $H_1 : \mu_1 - \mu_2 \neq 0$

Rejection Region:

$t > 1.782288$ or $t < -1.782288$

The test statistic:

$t = -0.0271$

Conclusion:

Do not Reject the null hypothesis.

We conclude based on the sample data there is no sufficient evidence to say there is a difference between the mean yields

Q4: A politician regularly polls her constituency to gauge her level of support among voters. This month, 652 out of 1158 voters support her. Five months ago, 412 out of 982 voters supported her. With a 5% significance level, can she infer that support has increased by at least 10 percentage points?

Answer:

The hypotheses are:

$$H_0 : p_1 - p_2 = .10$$

$$H_1 : p_1 - p_2 > .10$$

Rejection Region:

$$z > z_{.05} = 1.645$$

The test statistic: $\bar{p}_1 = \frac{652}{1158} = 0.56304$, $\bar{p}_2 = 0.41955$

$$Z_{cal} = \frac{\bar{p}_1 - \bar{p}_2 - 0.1}{\sqrt{\frac{\bar{p}_1 \bar{q}_1}{n_1} + \frac{\bar{p}_2 \bar{q}_2}{n_2}}} = 2.02674$$

Conclusion:

Reject the null hypothesis.

Based on the sample data, there is a sufficient evidence to say that her support has increased by at least 10 percentage points

Q5:

Weinberger and Spotts compare the use of humor in television ads in the United States and the United Kingdom. Suppose that independent random samples of television ads are taken in the two countries. A random of 400 television ads in the United Kingdom reveals that 142 use humor, while a random sample of 500 television ads in the United States reveals that 122 use humor.

- Set up the null and the alternative hypotheses needed to determine whether the proportion of ads using humor in the United Kingdom differs from the proportion of ads using in the United States.
- Test the hypotheses you set up in part (a) by using the test statistic approach and by using 0.01 level of significance.
- Set up the hypotheses needed to attempt to establish that the difference between the proportions of U.K and U.S ads using humor is more than 0.05. Test the hypotheses by using the P- value approach and by setting α equal to 0.01.

a) The hypotheses are:

$$H_0 : p_1 - p_2 = 0$$

$$H_1 : p_1 - p_2 \neq 0$$

b) Rejection Region:

$$z > z_{.005} = 2.575 \text{ or } z < -z_{.005} = -2.575$$

The test statistic: $\bar{p}_1 = \frac{142}{400} = 0.355$, $\bar{p}_2 = 0.244$

$$\bar{p} = \frac{122 + 142}{900} = 0.2933$$

$$Z_{cal} = \frac{\bar{p}_1 - \bar{p}_2 - 0}{\sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = 3.634366$$

Conclusion:

Reject the null hypothesis.

Based on the sample data, there is a sufficient evidence to say that there is a difference between the two proportions.

Answer for Part c:

c)

The hypotheses are:

$$H_0 : p_1 - p_2 = .05$$

$$H_1 : p_1 - p_2 > .05$$

Rejection Region:

Reject H_0 if the p -value less than $\alpha = 0.01$

The test statistic: $\bar{p}_1 = \frac{142}{400} = 0.355$, $\bar{p}_2 = 0.244$

$$Z_{cal} = \frac{\bar{p}_1 - \bar{p}_2 - 0.05}{\sqrt{\frac{\bar{p}_1 \bar{q}_1}{n_1} + \frac{\bar{p}_2 \bar{q}_2}{n_2}}} = 3.617794$$

p-value:

$$P_{val} = 2 * P(Z > 3.62) = 2 * (.5 - P(0 < Z < 3.62)) \approx 0.000$$

Conclusion:

Reject the null hypothesis.

Based on the sample data, there is a sufficient evidence to establish that the difference between the proportions of U.K and U.S ads using humor is more than 0.05.

Q6: The following information is available for two samples selected from independent populations:

Population G: $n = 16$ $S^2 = 47.3$

Population H: $n = 13$ $S^2 = 36.4$

Assume that two samples are selected from independent normally distributed populations.

- At the 0.05 level of significance, is there evidence of a difference in σ_1 and σ_2 ?
- Suppose that you want to perform a one-tail test. At the 0.05 level of significance, what is the upper-tail critical value of F to determine whether there is evidence that $\sigma_1 > \sigma_2$? What is your statistical decision?

(a) $H_0: \sigma_1^2 = \sigma_2^2$ The population variances are the same.

$H_1: \sigma_1^2 \neq \sigma_2^2$ The population variances are different.

Decision rule: If $F_{STAT} > 3.18$ reject H_0 .

$$\text{Test statistic: } F_{STAT} = \frac{S_1^2}{S_2^2} = \frac{47.3}{36.4} = 1.2995$$

Decision: Since $F_{STAT} = 1.2995$ is less than $F_{\alpha/2} = 3.18$, do not reject H_0 . There is not enough evidence to conclude that the two population variances are different.

(b) $H_0: \sigma_1^2 \leq \sigma_2^2$ The variance for population 1 is less than or equal to the variance for population 2.

$H_1: \sigma_1^2 > \sigma_2^2$ The variance for population 1 is greater than the variance for population 2.

Decision rule: If $F_{STAT} > 2.62$, reject H_0 .

$$\text{Test statistic: } F_{STAT} = \frac{S_1^2}{S_2^2} = \frac{47.3}{36.4} = 1.2995$$

Decision: Since $F_{STAT} = 1.2995$ is less than the critical bound of $F_{\alpha} = 2.62$, do not reject H_0 . There is not enough evidence to conclude that the variance for population 1 is greater than the variance for population 2.
