CH12 (More Problems)

1. A random sample of n = 500 observations were allocated to the k = 5 categories shown in the table. Suppose we want to test the null hypothesis that the category probabilities are p_1 =.1, p_2 =.1, p_3 =.5, p_4 =.1, and p_5 =.2.

Category					Total
1	2	3	4	5	
27	62	241	69	101	500

a. Calculate the expected cell counts.

b. Find χ^2_α for $\alpha = .05$.

c. State the alternative hypothesis for the test.

d. Do the data provide sufficient evidence to indicate that the null hypothesis is false?

(E2_053) The past output of a machine indicates that each unit it produces will be

Top grade	**With probability 0.38**
High grade	**With probability 0.32**
Medium grade	**With probability 0.291**
Low grade	**With probability 0.009**

A new machine, designed to perform the same job, has produced 500 items with the following results.

Grade	**Frequency (o_i)**	**Probability (p_i)**	**$e_i = n*p_i$**	
Top grade	206	**0.380**	**190**	
High grade	171	**0.320**	**160**	
Medium grade	120	**0.291**	**`145.5**	
Low grade	3	**0.009**	**4.5**	**Less than 5**
TOTAL	**500**	**1.00**	**500**	

Using 5% level of significance, **Can the difference in output be ascribed solely to chance**? Explain?

Solution:

Since the expected freq. for Low grade is less than 5 we combine the last two categories.

Grade	**Frequency (o_i)**	**Probability (p_i)**	**$e_i = n*p_i$**	$\frac{(o_i - e_i)^2}{e_i}$
Top grade	206	**0.380**	**190**	**1.347368421**
High grade	171	**0.320**	**160**	**0.75625**
Med & Low rgade	123	**0.300**	**`150**	**4.86**
TOTAL	**500**	**1.00**	**500**	**6.963618421**

1. The hypotheses:

H_0: There is no difference in output between the old and the new machine.

(The new machine will produced units with the same probability as the old machine)

(The difference ascribed solely to chance)

H_A: There is a difference in output between the old and the new machine.

(The new machine will <u>NOT</u> produced units with the same probability as the old machine)

(The difference can not be ascribed to chance)

1. **The test statistic**:

$$\chi^2_{cal} = \sum \frac{(o_i - e_i)^2}{e_i} = 6.963618421$$

2. **The critical value:**

d.f = 2

$$\chi^2_{\alpha,k-1} = \chi^2_{0.05,2} = 5.9915$$

3. **The decision Rule:**

Reject H_0 if $\chi^2_{cal} > \chi^2_{\alpha,k-1}$, otherwise don't reject H_0

4. **Decision:**

Since 6.963618421 > 5.9915, reject H_0

6. **Conclusion:**

There is a difference in output between the old and the new machine.

2. Refer to the accompanying 2×3 contingency table.

		Columns 1	2	3	Totals
Rows	1	14	37	23	74
	2	21	32	38	91
Totals		35	69	61	165

a. Calculate the estimated expected cell counts for the contingency table.

b. Calculate the chi-square statistic for the table.

3. The management of a certain hotel is interested in whether all its guests are treated the same regardless of the prices of their rooms. They randomly chose 165 recent guests and questioned them about the service they had received at the hotel. The following summary data resulted:

Service ranking	Type of room			
	Economy	Standard	Luxury	
Excellent	30	21	9	
Good	36	29	8	
Fair	12	8	12	

What conclusion would you draw? (Use the 10% level of significance, and show your work in details)