

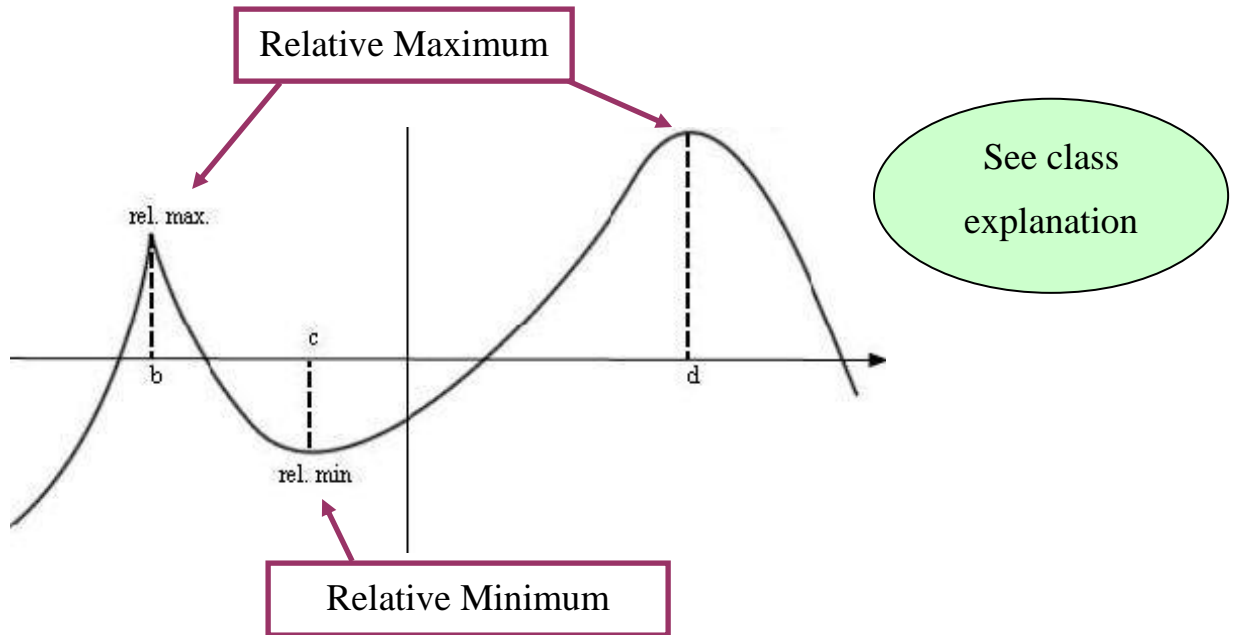
**Learning outcomes**

After completing this section, you will inshaAllah be able to

1. explain what is meant by **relative (local) maximum and minimum**
2. explain what is meant by **absolute maximum and minimum**
3. explain the meaning of **increasing or decreasing intervals** of a function
4. explain what are **critical values**
5. find critical values
6. **determine absolute extrema** of a function **on a closed interval**

## What are relative (local) extreme points?

- Graphical explanation



- Given a function  $f(x)$ . It has

**Relative maximum** at  $x_0$  if  
 $f(x_0) \geq f(x)$  for all points  $x$  **near**  $x_0$ .

Highest of  
points near  $x_0$ .

**Relative minimum** at  $x_0$  if  
 $f(x_0) \leq f(x)$  for all points  $x$  **near**  $x_0$ .

Lowest of  
points near  $x_0$ .

## What are absolute extreme points?

- Given a function  $f(x)$ . It has

**Absolute maximum** at  $x_0$  if

$$f(x_0) \geq f(x) \text{ for all points } x \text{ in domain.}$$

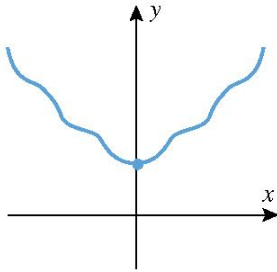
Highest of all  
points in domain

**Absolute minimum** at  $x_0$  if

$$f(x_0) \leq f(x) \text{ for all points } x \text{ in domain.}$$

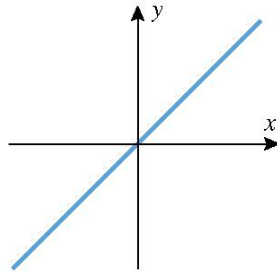
Lowest of all  
points in domain

- Some examples of absolute extreme points



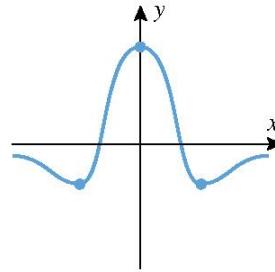
$f$  has an absolute minimum but no absolute maximum on  $(-\infty, +\infty)$ .

(a)



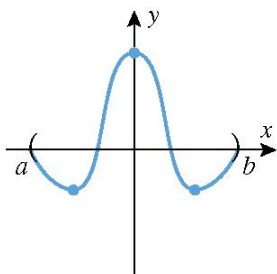
$f$  has no absolute extrema on  $(-\infty, +\infty)$ .

(b)



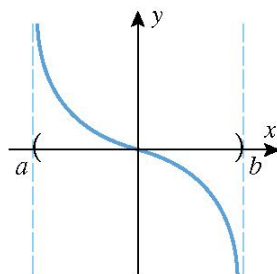
$f$  has an absolute maximum and minimum on  $(-\infty, +\infty)$ .

(c)



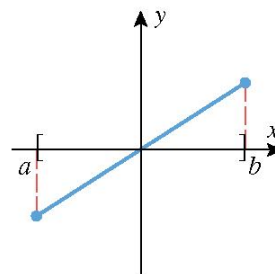
$f$  has an absolute maximum and minimum on  $(a, b)$ .

(d)



$f$  has no absolute extrema on  $(a, b)$ .

(e)



$f$  has an absolute maximum and minimum on  $[a, b]$ .

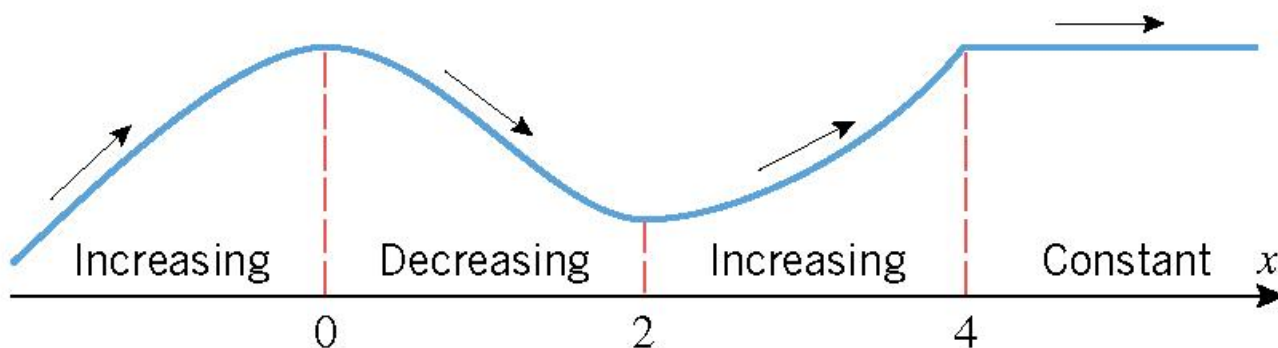
(f)

In general, a function may not have absolute maximum or absolute minimum or both.

See example 1 done in class

## Increasing or decreasing functions

- Graphical explanation.



- Given a function  $f(x)$ . Then

$$f' > 0 \text{ on } I \Rightarrow f \text{ is increasing on } I$$

$$f' < 0 \text{ on } I \Rightarrow f \text{ is decreasing on } I$$

$$f' = 0 \text{ on } I \Rightarrow f \text{ is constant on } I$$

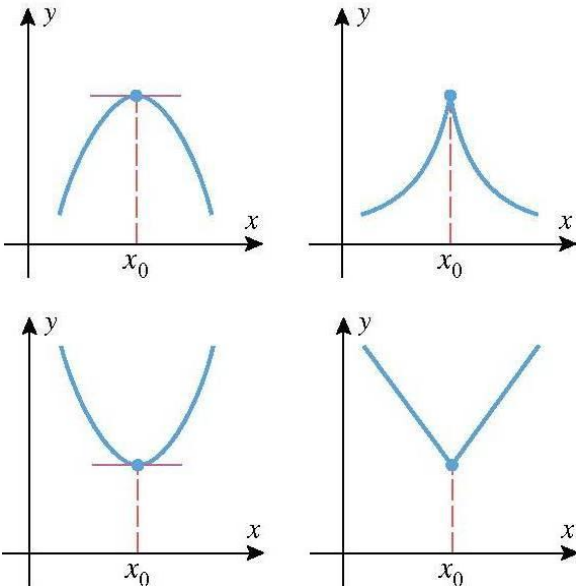
See class  
explanation

- Next we look at two important questions about increasing/decreasing functions.
- These questions will lead us to understand the concept of critical values.

## Two important questions regarding increasing or decreasing intervals

**Q.1.** At which points can a function change from increasing to decreasing or decreasing to increasing?

Obviously at points where  $f'$  changes sign. Why?



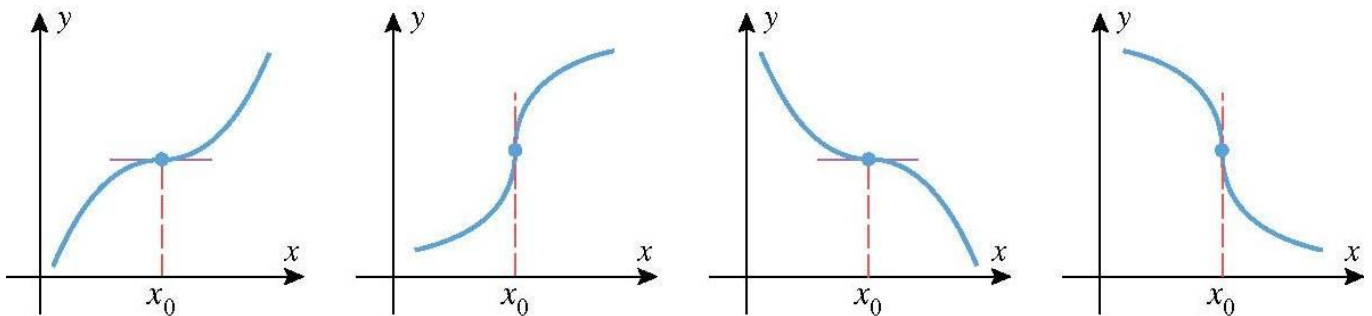
A function **can** change from increasing to decreasing or decreasing to increasing **ONLY at points where  $f' = 0$  or  $f'$  undefined.**

- See class explanation

**Q.2.** At such points will the function surely change from increasing to decreasing or decreasing to increasing?

**No. It will not happen always.**

- See class explanation and the graphical explanation below.



## How is the concept of increasing/decreasing functions related to the concept of relative extreme points?

- We can also look at relative extreme points from the following view.

Relative **maximum** point: where  $f$  changes from **increasing to decreasing**

Relative **minimum** point: where  $f$  changes from **decreasing to increasing**

- Before we learn how to find relative extrema, recall from previous page
  - **Relative extrema can occur only** at points where  $f' = 0$  or  $f'$  undefined
  - But it is **not necessary that all such points will be relative extreme points**. **We must check further** to see if “ $f$  changes from increasing to decreasing or decreasing to increasing at these points”.

### Critical values

- Given a function  $f(x)$

The values of  $x$  where  $f' = 0$  or  $f'$  undefined are called **critical values** of  $f(x)$

- These are **possible relative extreme points**
- Relative extrema can only occur at these points
- But we **must check further**

See example 2, 3, 4 done in class

Usually

useful

when

4.1<sub>7</sub>

## Absolute extrema on a closed interval

- Recall from 4.1<sub>3</sub> that in general a function may not have absolute extrema. But for closed interval we have a definite answer.

### Extreme value theorem

A continuous function has an absolute maximum and an absolute minimum on a **closed interval**

See graphical explanation in class

### To find absolute extrema of $f(x)$ on $[a,b]$

- Find critical values of  $f(x)$  in the interval
- Evaluate  $f(x)$  at all critical values
- Evaluate  $f(x)$  at end points of interval
- The largest value (of above) is absolute maximum  
The smallest value (of above) is absolute minimum

See class explanation

See examples 5, 6 done in class

*End of 4.1*