Learning outcomes

After completing this section, you will inshaAllah be able to

- 1. know the hyperbolic functions
- 2. understand fundamental hyperbolic identities
- 3. differentiate hyperbolic & inverse hyperbolic functions

Definition of hyperbolic functions

• Hyperbolic functions are defined using e^x and e^{-x}

The six hyperbolic functions are defined as

$$\sinh x = \frac{e^x - e^{-x}}{2},$$

$$\operatorname{csch} x = \frac{2}{e^x - e^{-x}}$$

$$\cosh x = \frac{e^x + e^{-x}}{2},$$

$$\operatorname{sech} x = \frac{2}{e^x + e^{-x}}$$

$$\tanh x = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}},$$

$$\coth x = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

See examples 1, 2 done in class

Fundamental hyperbolic identities

- Like trigonometric identities, the hyperbolic identities are very useful in simplifying expressions and questions involving differentiation, integration etc.
- The fundamental hyperbolic identities are as follows.

$ \cosh x + \sinh x = e^x, $	$ \cosh x - \sinh x = e^{-x} $
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$$\cosh(-x) = \cosh x , \qquad \sinh(-x) = -\sinh x$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$1 - \tanh^2 x = \operatorname{sec} h^2 x$$

$$\sinh 2x = 2\sinh x \cosh x,$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\cosh 2x = 2\sinh^2 x + 1$$

$$\cosh 2x = 2\cosh^2 x + 1$$

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\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y
\sinh(x - y) = \sinh x \cosh y - \cosh x \sinh y
\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y
\cosh(x - y) = \cosh x \cosh y - \sinh x \sinh y
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All of the above and similar identities can easily be proved by using the definitions of hyperbolic functions.

See examples 3, 4 done in class

Derivatives of hyperbolic functions

• The differentiation formulas for hyperbolic functions can easily be derived by differentiating the R.H.S. of definition of hyperbolic functions.

Differentiation formulas for hyperbolic functions
$$\frac{d}{dx}(\sinh u) = \cosh u \frac{du}{dx}$$

$$\frac{d}{dx}(\cosh u) = \sinh u \frac{du}{dx}$$

$$\frac{d}{dx}(\tanh u) = \operatorname{sech}^{2} u \frac{du}{dx}$$

$$\frac{d}{dx}(\coth u) = -\operatorname{csch}^{2} u \frac{du}{dx}$$

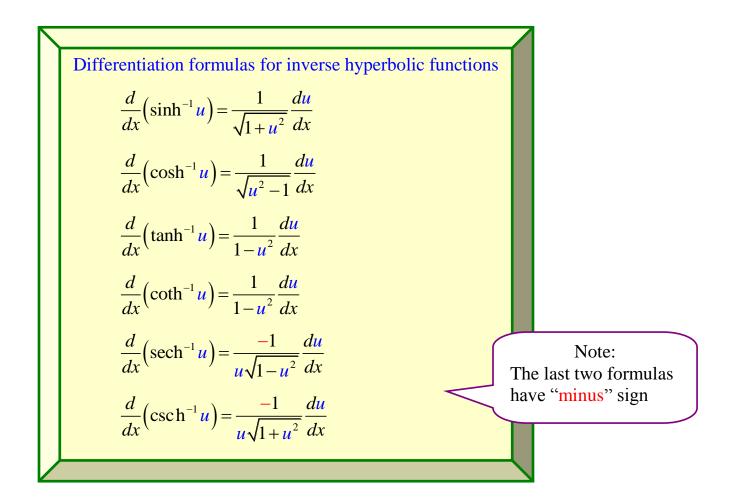
$$\frac{d}{dx}(\operatorname{sech} u) = -\operatorname{sech} u \tanh u \frac{du}{dx}$$

$$\frac{d}{dx}(\operatorname{csch} u) = -\operatorname{csch} u \cot u$$

See examples 5, 6 done in class

Derivatives of inverse hyperbolic functions

 Derived from the differentiation formulas of hyperbolic functions and using implicit differentiation. See class explanation for more.



See example 7, 8 done in class