

Learning outcomes

After completing this section, you will inshaAllah be able to

1. use a special type of **limit to find slopes of tangent lines**
2. use a special type of **limit to find rate of change of a function**
3. explain the **definition of derivative** of a function at a point
4. use a special type of **limit to find derivative at a point**

Slope of tangent line to a curve $y=f(x)$

- Given a curve $y = f(x)$

The **slope** of tangent line at $(a, f(a))$ is given by

$$m = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

See class explanation

See example 1 done in class

If we take $x = a + h$ then the above definition becomes

The **slope** of tangent line at $(a, f(a))$ is given by

$$m = \lim_{h \rightarrow 0} \frac{f(a + h) - f(a)}{h}$$

See class explanation

See example 2 done in class

Instantaneous rate of change

- Given a function $y = f(x)$

Average rate of change of $f(x)$ over $[a, a+h]$

$$y_{av} = \frac{\text{change in } f(x)}{\text{change in } x} = \frac{f(a+h) - f(a)}{h}$$

Instantaneous rate of change in $f(x)$ at $x = a$

$$= \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

See class
explanation

Every day example

$S=f(t)$: position of object at time t

Average velocity v_{av} over interval $[a, a+h]$

= average rate of change of displacement over interval

$$\Rightarrow v_{av} = \frac{f(a+h) - f(a)}{h}$$

See example 3
done in class

Velocity at time $t =$

Instantaneous rate of change of displacement at time t

$$\Rightarrow v = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

An important remark

- Given a function $y = f(x)$.
- We have seen above that **limits of the form**

$$\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

are **very special** and can be used to answer important questions.

- For example, the **rate of change** of $f(x)$ at a point 'x' is given by

$$m = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

- Therefore **such limits become very important**.
- The study of important limit of above type **leads to the concept of derivative**, which we study next.

Usually

useful

when

2.7₅

Derivative of a function

The derivative of $f(x)$ at a point ' a ' is defined by

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h},$$

if the limit exists.

If we take $x = a + h$ then the above definition becomes

The derivative of $f(x)$ at a point ' a ' is defined by

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

if the limit exists.

See examples 4, 5 done in class

End of 2.7