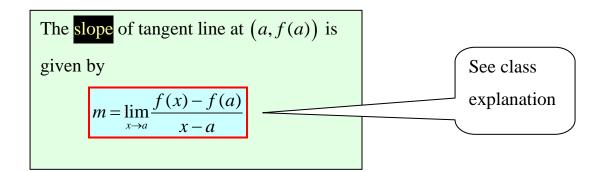
Learning outcomes

After completing this section, you will inshaAllah be able to

- 1. use a special type of limit to find slopes of tangent lines
- 2. use a special type of limit to find rate of change of a function
- 3. explain the definition of derivative of a function at a point
- 4. use a special type of limit to find derivative at a point

Slope of tangent line to a curve y=f(x)

• Given a curve y = f(x)



See example 1 done in class

If we take x = a + h then the above definition becomes

The slope of tangent line at (a, f(a)) is given by

 $m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

See class explanation

See example 2 done in class

Instantaneous rate of change

• Given a function y = f(x)

Average rate of change of f(x) over [a, a+h]

$$y_{av} = \frac{\text{change in } f(x)}{\text{change in } x} = \frac{f(a+h) - f(a)}{h}$$

Instantaneous rate of change in f(x) at x = a

$$= \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

See class explanation

Every day example

S = f(t): position of object at time t

Average velocity v_{av} over interval [a, a+h]

= average rate of change of displacement over interval

$$\Rightarrow v_{av} = \frac{f(a+h) - f(a)}{h}$$

See example 3 done in class

Velocity at time t =

Instantaneous rate of change of displacement at time t

$$\Rightarrow v = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

An important remark

- Given a function y = f(x).
- We have seen above that limits of the form

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

are very special and can be used to answer important questions.

• For example, the rate of change of f(x) at a point 'x' is given by

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

- Therefore such limits become very important.
- The study of important limit of above type leads to the concept of derivative, which we study next.

when

Derivative of a function

The derivative of f(x) at a point 'a' is defined by

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

if the limit exists.

If we take x = a + h then the above definition becomes

The derivative of f(x) at a point 'a' is defined by

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

if the limit exists.

See examples 4, 5 done in class