

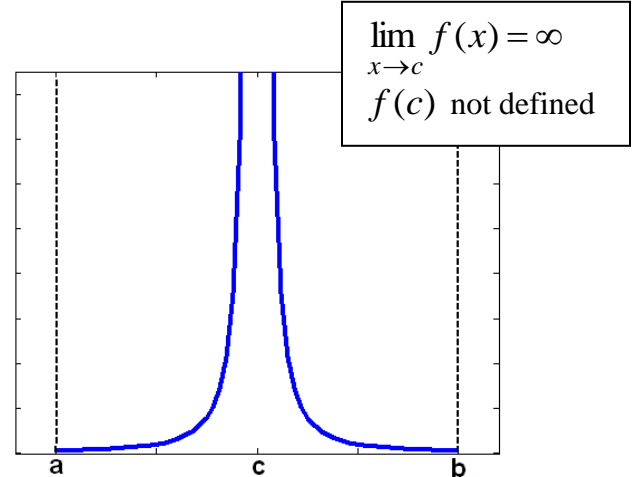
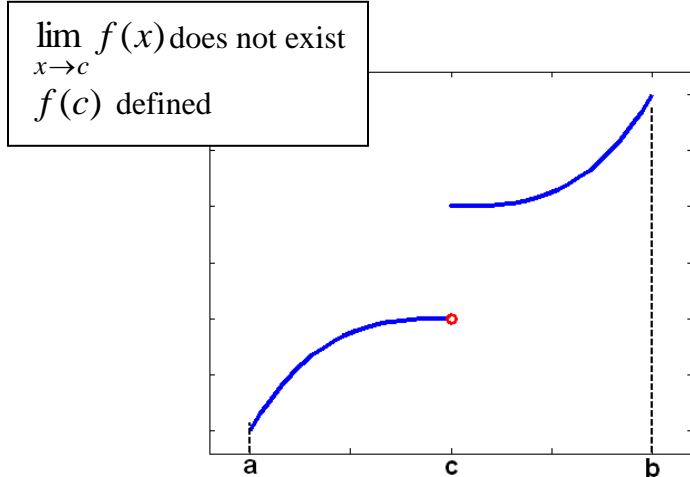
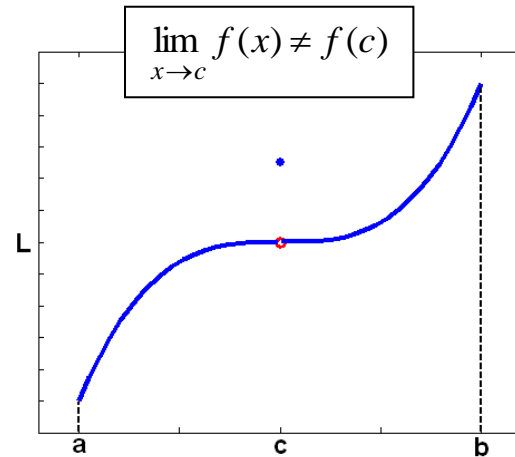
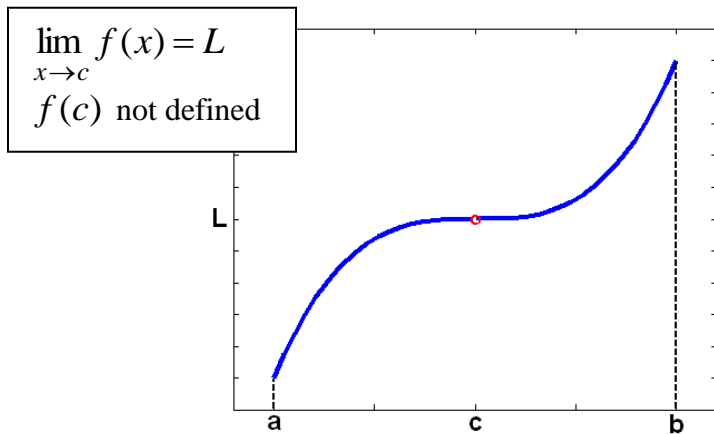
Learning outcomes

After completing this section, you will inshaAllah be able to

1. get an idea about the meaning of a **continuous function**
2. **check** whether a function is **continuous or discontinuous** at a point
3. use **basic properties** of continuous functions
4. know **important examples** of continuous functions
5. explain difference between **different types of discontinuities**
 - a. **removable discontinuity**
 - b. **jump discontinuity**
 - c. **infinite discontinuity**
6. explain and apply **intermediate value theorem**

Meaning of continuous function

- The following graphs have gaps. Let's see what is happening in these graphs.



Continuity \approx no gap(s) in the graph.

Clearly: To have continuity at $x=c$, none of above should happen

A function $f(x)$ is continuous at $x=a$ if $\lim_{x \rightarrow a} f(x) = f(a)$

i.e.

- $f(a)$ is defined
- $\lim_{x \rightarrow a} f(x)$ exists
- $\lim_{x \rightarrow a} f(x) = f(a)$

Computations related to continuity

To show $f(x)$ is continuous at $x=a$ we must show

1. $f(a)$ is defined
2. $\lim_{x \rightarrow a} f(x)$ exists
3. $\lim_{x \rightarrow a} f(x) = f(a)$

- left limit = right limit
- answer is a finite number

See examples 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 done in class

While doing questions we also keep using the following important facts, as needed.

Basic Properties

If f, g are continuous at point ' a ' then

1. $f \pm g$
2. $f \cdot g$
3. cf (c constant)
4. $\frac{f}{g}$

are also continuous at ' a '.

The composition $f \circ g$ of continuous functions f, g is also continuous

Important examples

- all **polynomials** are **continuous**
- **rational** functions, **root** functions, **trigonometric**, **inverse trigonometric**, **logarithmic**, **exponential** functions are **continuous** at points in their domain
- e.g. This means **rational functions** are **discontinuous** only at points where **denominator** is **zero**

What are different types of discontinuities that can occur?

- We learn the different types of discontinuities with the help of examples.

Removable discontinuity

See example 12 and
explanation provided
in class

Infinite discontinuity

See example 13 and
explanation provided
in class

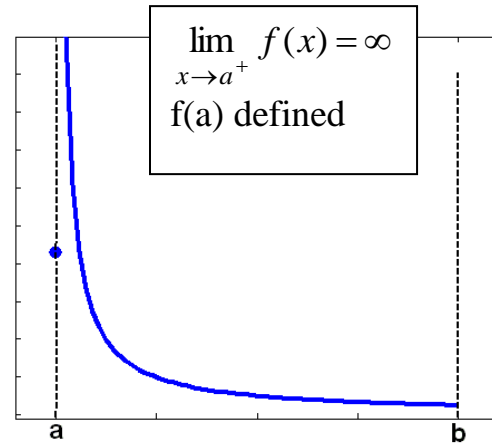
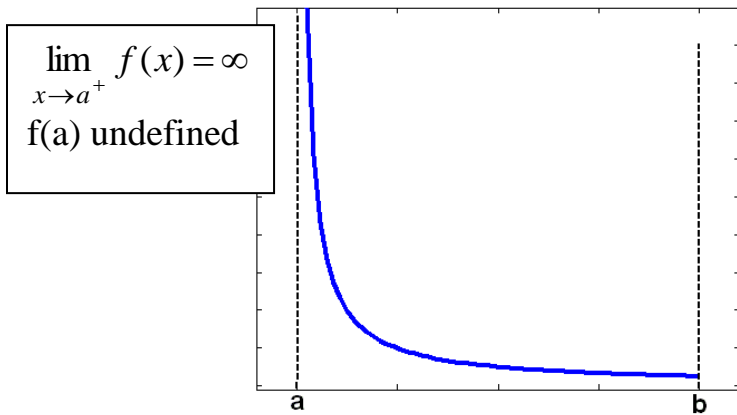
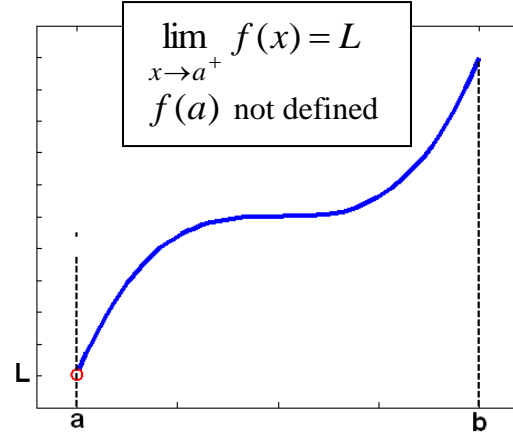
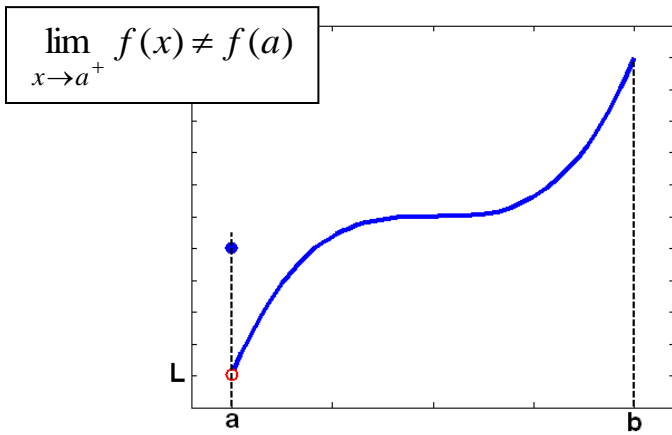
Jump discontinuity

See example 14 and
explanation provided
in class

See examples 15, 16 done in class

How to check continuity at end points of a closed interval [a,b]

- Look at following graphs for left end point $x=a$



Continuity \approx no gap(s) in the graph.

Clearly: To have continuity at $x=c$, none of above should happen

$f(x)$ is continuous at left end point $x=a$ of $[a,b]$ if $\lim_{x \rightarrow a^+} f(x) = f(a)$

i.e.

- $f(a)$ is defined
- $\lim_{x \rightarrow a^+} f(x)$ exists (i.e finite)
- $\lim_{x \rightarrow a^+} f(x) = f(a)$

Similarly

$f(x)$ is continuous at right end point $x=b$ of $[a,b]$ if $\lim_{x \rightarrow b^-} f(x) = f(b)$

Continuity on an interval

A function $f(x)$ is continuous on an interval if it is continuous at every point in the interval

If an end point is included in the interval, use the idea of **left-continuous or right-continuous** as appropriate.

See explanation on last page or definitions below

$f(x)$ is **continuous from right** at $x=a$ if

$$\lim_{x \rightarrow a^+} f(x) = f(a)$$

$f(x)$ is **continuous from left** at $x=b$ if

$$\lim_{x \rightarrow b^-} f(x) = f(b)$$

See example 17 done in class

Some applications**Limits of composition of continuous functions**

$$\lim_{x \rightarrow a} f(g(x)) = f\left(\lim_{x \rightarrow a} g(x)\right)$$

Very useful tool
for calculating
complicated limits

See example 18 done in class

Intermediate value theorem & applications

If $f(x)$ is continuous on $[a, b]$ and
 N is any number between $f(a)$ and $f(b)$
then
there is a number c in (a, b) such that $f(c) = N$

See class
explanation

See examples 19, 20, 21 done in class

End of 2.5