Section 2.4 *The precise definition of a limit*

(only problems like Examples 1, 2 are in the syllabus)

Learning outcomes

After completing this section, you will inshaAllah be able to

1. understand the precise definition of limit
2. use the definition of limit to study limits of some functions

Formal definition of limit

- Look at example of Page 2.2_2 and recall from Page 2.3_2.

Informally $\lim\limits_{x \to a} f(x) = L$

if we can make $f(x)$ as close to L as we like

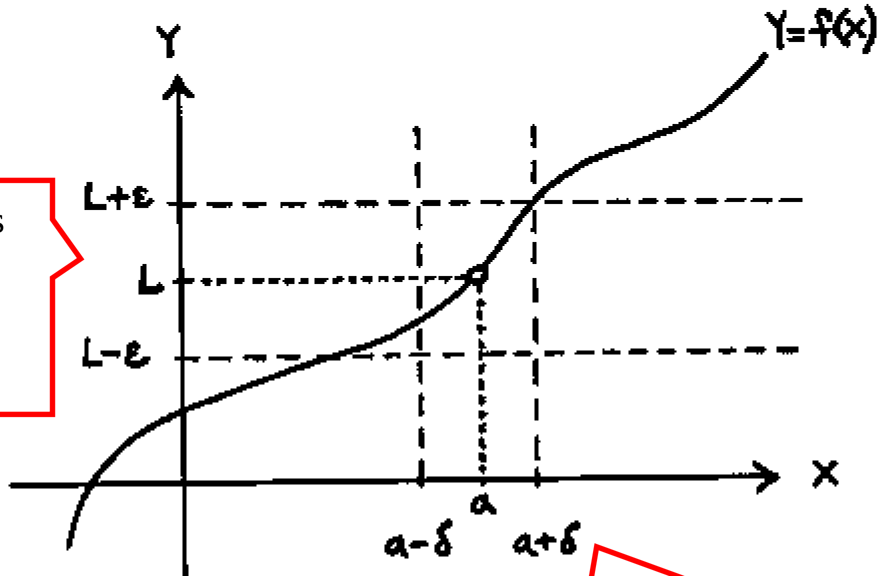
by taking x sufficiently close to a

- Formally we state this as

Formally $\lim\limits_{x \to a} f(x) = L$ if

for every given number $\varepsilon > 0$, we can find a number $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $0 < |x - a| < \delta$



Recall $|f(x) - L| < \varepsilon$ means

$-\varepsilon < f(x) - L < \varepsilon$

or $L - \varepsilon < f(x) < L + \varepsilon$

Similarly $|x - a| < \delta$ means

$-\delta < x - a < \delta$

or $a - \delta < x < a + \delta$

See examples 1, 2 done in class

Usually useful when

Strategy for proving $\lim_{x \to a} f(x) = L$ using definition

What to do?

- Consider any number $\varepsilon > 0$.
- We are required to show that we can find a number $\delta > 0$ such that

$$0 < |x - a| < \delta \quad \Rightarrow \quad |f(x) - L| < \varepsilon$$

This is usually done through following two main steps

- Analyzing $|f(x) - L| < \varepsilon$ to make a choice for δ.

  This involves starting with the expression $|f(x) - L|$ and simplifying it reach the expression involving $|x - a|$.

- Using the chosen δ to formally prove the limit by showing that

$$0 < |x - a| < \delta \quad \Rightarrow \quad |f(x) - L| < \varepsilon$$

See example 3 done in class

Do exercises given in class

End of 2.4