

Question

Find approximate value of $(8.06)^{\frac{2}{3}}$.

Solution

- Consider $f(x) = x^{\frac{2}{3}}$
 - The question is to approximately find $f(8.06)$
- We will use the idea of local linear approximation which is

Linear Approximation of $f(x)$

For values of x near $x=a$

$$f(x) \approx f(a) + f'(a)(x-a)$$

- In our case we are interested in knowing $f(x)$ near $x=8$ so we have

For values of x near $x=8$

$$f(x) \approx f(8) + f'(8)(x-8)$$

- So we can use the following equation to approximate $f(8.06)$

$$f(8.06) \approx f(8) + f'(8)(8.06-8) \quad (*)$$

- Now

$$f(x) = x^{\frac{2}{3}} \quad \Rightarrow \quad f(8) = 4$$

$$f'(x) = \frac{2}{3x^{\frac{1}{3}}} \quad \Rightarrow \quad f'(8) = \frac{1}{3}$$

- Using in Equation (*) gives

$$f(8.06) \approx 4 + \frac{1}{3}(.06) = 4.02$$