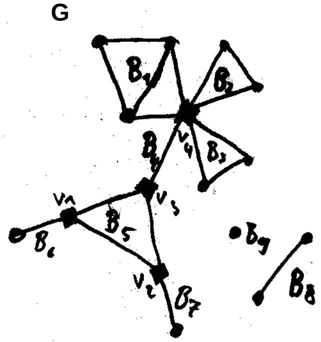
The Block Graph

In a sense, blocks are the 2-connected analogues of components, the maximal connected subgraphs of a graph. While the structure of G is determined fully by that of its components, however, it is not captured completely by the structure of its blocks:
since the blocks need not be disjoint, the way they intersect defines another structure, giving a coarse picture of G as if viewed from a distance.

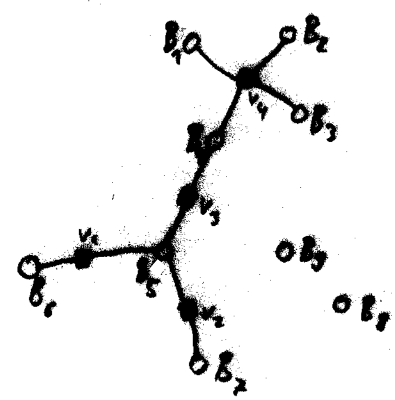
The following theorem describes this coarse structure of G as formed by its blocks. Let A denote the set of cutvertices of G, and $\mathcal{B}$ the set of its blocks. We then have a natural bipatite graph on $A \cup \mathcal{B}$ formed by the edges aB with $A \ni a \in B \in \mathcal{B}$. This <u>"block graph"</u> of G is shown below:

G



cut-vertices
v_1, v_2, v_3, v_4

Block Graph of G



<u>Theorem 2.A:</u>

The block graph of a connected graph is a tree, i.e., it is connected and does not contain a cycle.

<u>Proof:</u>

Assume there would be a cycle $v_1 B_1, v_2, B_2, \dots, v_e, B_e, v_{e+1} = v_1$ inside the block graph, then $C := B_1 \cup B_2 \cup \dots \cup B_e$ (where $B_1 \cap B_2 = \{v_2\}, \dots$) is inseperable (it does not contain cut-vertices with respect to itself).
Therfore $C \supsetneq B_1$, B_1 is not maximal inseperable (no block). ↯