# 12.7 Cylindrical and Spherical Coordinates

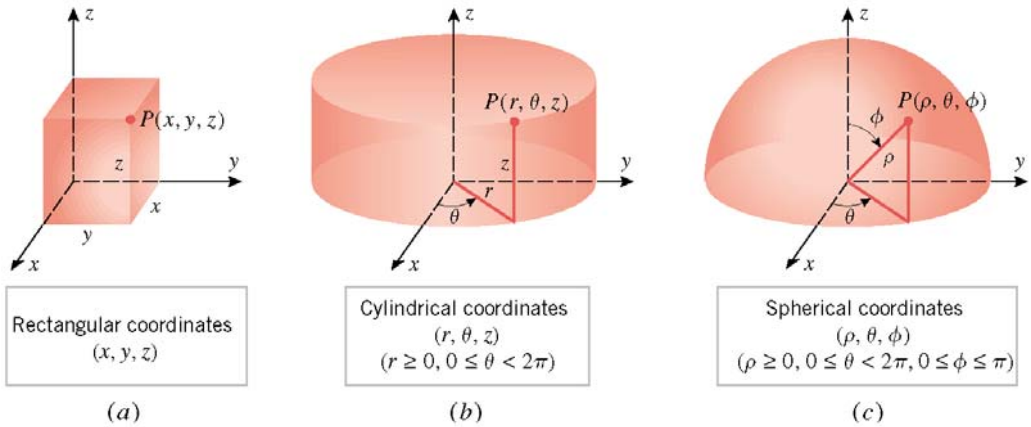
- In 2-dimensions, we learnt polar coordinates which gave an easier description of some curves.
- Here, we introduce two coordinate systems in 3-dimensions, known as cylindrical coordinate system and spherical coordinate system which give easier description of some surfaces.

In 2-space, every point can be represented by two ways: one is in rectangular coordinate system and other is in polar coordinate system. In 3-space each point can be represented by three ways:

***(i) rectangular coordinates***

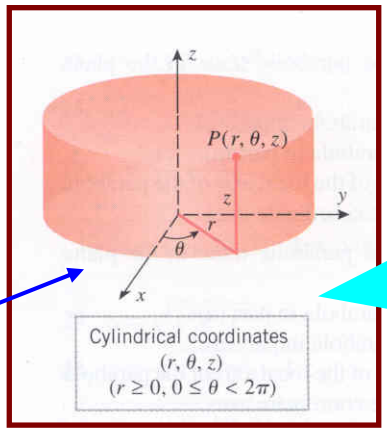
***(ii) cylindrical coordinates and***

***(iii) spherical coordinates.***



## What are cylindrical coordinates?

A point $P$ in 3-space represented by coordinates $(r,\theta,z)$, where $r, \theta$ and $z$ are as shown in the figure




Restriction: $r \geq 0$ and $0 \leq \theta < 2\pi$

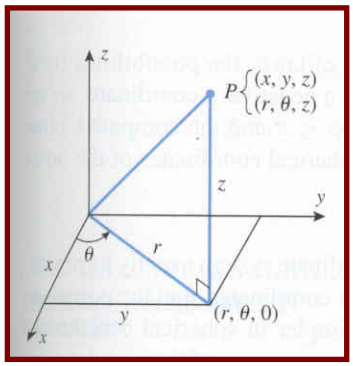
## Relation between cylindrical & rectangular coordinates

- Cylindrical to rectangular: $(r,\theta,z) \rightarrow (x,y,z)$

$$x = r\cos\theta, \quad y = r\sin\theta, \quad z = z$$

- Rectangular to cylindrical: $(x,y,z) \rightarrow (r,\theta,z)$

$$r = \sqrt{x^2 + y^2}, \quad \tan\theta = \frac{y}{x}, \quad z = z$$




**Question 10/843:** Convert the rectangular coordinates $(3,3,-2)$ to the cylindrical coordinates.

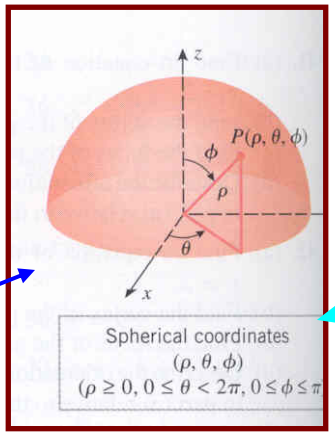
**Question 4/843:** Plot the point whose cylindrical coordinates are $\left(1, 3\pi/2, 2\right)$ and **then find the rectangular coordinates of the point.**

## What are spherical coordinates?

See class explanation

A point $P$ in 3-space represented by coordinates $(\rho,\theta,\phi)$, where

- $\rho$ is distance of $P$ from origin
- $\theta$, $\phi$ as shown in the figure




Restriction:

$\rho \geq 0$ and

$0 \leq \theta < 2\pi$

$0 \leq \phi \leq \pi$

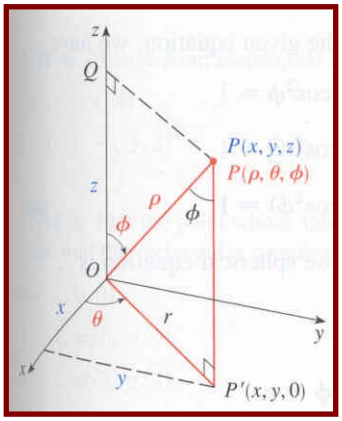
## Relation between spherical & rectangular coordinates

- Spherical to rectangular: $(\rho,\theta,\phi) \rightarrow (x,y,z)$

$$x = \rho\sin\phi\cos\theta, \quad y = \rho\sin\phi\sin\theta, \quad z = \rho\cos\phi$$

- Rectangular to spherical: $(x,y,z) \rightarrow (\rho,\theta,\phi)$

$$\rho = \sqrt{x^2+y^2+z^2}, \quad \tan\theta = \frac{y}{x}, \quad \cos\phi = \frac{z}{\sqrt{x^2+y^2+z^2}}$$




**Question 17/843**: Plot the point whose spherical coordinates are $(2,\pi/3,\pi/4)$ and convert into rectangular coordinates.

**Question 22/843**: Change $(-1,1,\sqrt{6})$ from rectangular to spherical coordinates.

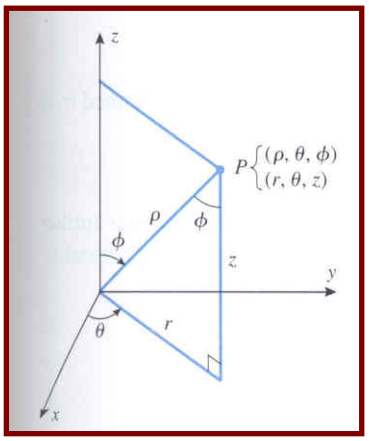
# Conversion between cylindrical & spherical coordinates

- Spherical to cylindrical: $(\rho,\theta,\phi)\to(r,\theta,z)$

$$r=\rho\sin\phi,\quad \theta=\theta,\quad z=\rho\cos\phi$$

- Cylindrical to Spherical: $(r,\theta,z)\to(\rho,\theta,\phi)$

$$\rho=\sqrt{r^2+z^2},\quad \theta=\theta,\quad \tan\phi=\frac{r}{z}$$




**Question 40/843:** Identify the surface $\rho\sin\phi=2$.

**Question 45/843:** Identify the surface $\rho^2(\sin^2\phi\cos^2\theta+\cos^2\phi)=4$.

**Question 62/843:** Sketch the solid described by the inequalities

$$-\pi/2\le\theta\le\pi/2,\quad 0\le\phi\le\pi/6,\quad 0\le\rho\le\sec\phi.$$