KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICAL SCIENCES
DHAHRAN, SAUDI ARABIA

STAT 211 (A): BUSINESS STATISTICS I

Semester 052
Major Exam #2
Wednesday April 26, 2006

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Name: ID# Section: Serial:

Question No	Full Marks	Marks Obtained
1	10	
2	13	
3	10	
4	15	
5	5	
6	12	
Total	65	

Question .1(*4+6=10-Points*)

Ahmed followed stock exchanges over the past 50 days. In particular, he recorded price exchanges for two stocks, Al- Kahraba' and Safco. He partially constructed the following table.

		Safco price↓			Total
		Decrease	Unchanged	Increase	
Al-Kahraba ' price →	Decrease	50	10	60	120
	Unchanged	70	*50*	80	200
	Increase	*70*	40	*70*	180
Total		190	100	*210*	500

With this method, he partially completed the following table to study the behavior of the two stocks.

		Safco↓			Total
		Decrease	Unchanged	Increase	
Al-Kahraba ' →	Decrease	0.10	0.02	0.12	0.24
	Unchanged	0.14	*0.10*	0.16	0.40
	Increase	*0.14*	0.08	*0.14*	0.36
Total		0.38	0.20	*0.42*	1

With this partially constructed table, find the following:

a. What is the probability that Al-Kahraba' stock price decreases given that Safco price decreases?

$$P(\text{Al-Kahraba' decrease}\mid\text{Safco Price decrease}) = \frac{P(\text{both decrease})}{P(\text{Safco decrease})} \quad (1pt)$$
$$= \frac{0.10}{0.38} \quad (1pt)$$
$$= 0.263158 \quad (1pt)$$

b. Let A: Safco increases and B: Al-Kahraba' stock increases.

I. Are these two events mutually exclusive? Why?

No (1pt). $P(A\cap B)=0.14$ (by the complement rule)

$P(A\cap B)\neq 0$. So, events A and B are <u>NOT mutually exclusive</u> (1pt)

II. Are these two events independent? Why?

$P(A\cap B)=0.14$ No.(1pt)

$P(A)=0.42$ $P(A)P(B) =0.42(0.36)$

$P(B)=0.36$ $=0.1512$

$P(A\cap B)\neq P(A)P(B)$ (1pt). So, <u>not independent</u>.

III. Find $P(A\cup B)$

$P(A\cup B) = P(A)+P(B)-P(A\cap B)$ (Probability Addition rule)

$=0.42+0.36-0.14$ (1pt)

$=0.64$ (1pt)

Question .2 (*1+2+4+2+4=13-Points*)

The following distribution of number of daily customer complaints was observed for the past year at Giant Supermarket

X	0	1	2	3	4	5
P(X)	0.2	0.25	0.25	0.1	0.1	0.1

a. What type of probability distribution is represented above?

Discrete Probability Distribution (1pt)

b. Find the probability that on a given day, there will be at least one customer complaint?

$P(x \geq 1) = 1 - P(x = 0)$

$= 1 - 0.20$ (1pt)

$= 0.80$ (1pt)

c. Find the probability that on a given day, there will be between 2 and 4 complaints (inclusive) given that there is at least one complaint.

$$P(2 \leq x \leq 4 \mid x \geq 1) = \frac{P(2 \leq x \leq 4 \text{ and } x \geq 1)}{P(x \geq 1)} \quad (1pt)$$

$$= \frac{P(x=2)+P(x=3)+P(x=4)}{1-P(x=0)} \quad (1pt)$$

$$= \frac{0.25+0.10+0.10}{0.80} \quad (1pt)$$

$$= \frac{0.45}{0.80}$$

$$= 0.5625 \quad (1pt)$$

d. Find the expected number of customer complaints?

$E[x] = \sum xP(x)$

$= 0(0.20)+1(0.25)+2(0.25)+3(0.10)+4(0.10)+5(0.10)$ (1pt)

$= 0+0.25+0.50+0.30+0.40+0.50$

$= 1.95$ (1pt)

e. Find the standard deviation of customer complaints?

(1pt) This method

X	P(X)	XP(X)	$X^2P(X)$	or	$(X-E[X])^2P(X)$
0	0.20	0.00	0.00		0.760500
1	0.25	0.25	0.25		0.225625
2	0.25	0.50	1.00		0.000625
3	0.10	0.30	0.90		0.110250
4	0.10	0.40	1.60		0.420250
5	0.10	0.50	2.50		0.930250
	1.00	1.95	6.25		2.4475
			(1pt)		(or 1pt)

$$\sigma = \sqrt{\sum x^2 P(x) - (E[X])^2}$$

$$= \sqrt{6.25 - (1.95)^2}$$

$$= \sqrt{2.4475} \quad (1pt)$$

$$= 1.56445 \quad (1pt)$$

Question .3(*3+4+3=10-Points*)

The life time of batteries manufactured by a factory has an exponential distribution with mean 320 hours. A battery is selected randomly from the product of the factory. Then:

a. Find the probability that the battery will work at most 300 hours.

$P\left(x \le 300\right) = P(0 \le x \le 300)$

$= 1 - e^{-\lambda(300)}$ but what is λ? exponential mean $=1/\lambda$

320 hr $=1/\lambda$. So, $\lambda = 1/320$ (1 pt)

$= 1 - e^{-300/320}$

$= 1 - e^{-0.9375} \quad (1pt)$

$= 0.608394 \quad (1pt)$

b. Find the probability that the battery will work more than 400 hours given that it has worked more than 360 hours.

$P\left(x > 400 \,|\, x > 360\right) = \frac{P(x > 400 \cap x > 360)}{P(x > 360)} \quad (1pt)$

$= \frac{1-\left(1-e^{-\lambda 400}\right)}{1-\left(1-e^{-\lambda 360}\right)}$

$= \frac{e^{-400/320}}{e^{-360/320}} \quad (1pt)$

$= \frac{0.286505}{0.324652}$

$= 0.882497 \quad (1pt)$

c. Find the median of the life time of the battery.

Exponential is skewed so mean is not median

$P\left(0 \le x \le median\right) = 0.50 \quad (1pt)$

$1 - e^{-\lambda(median)} = 0.50$

$1 - e^{-median/320} = 0.50$

$e^{-median/320} = 0.50$

$\frac{-median}{320} = ln(0.50)$

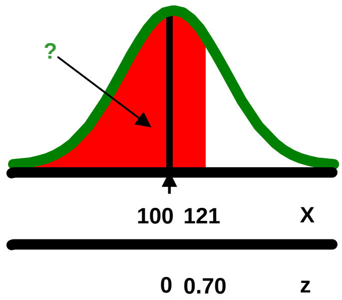
$Median = -320 ln(0.50)$ (1pt)

$= 221.807 \quad (1pt)$

<u>Question .4(*3+4+4+4=15-Points*)</u>

At KFUPM the distribution of student after-class daily studying time has been known to follow a ***normal distribution*** with a ***mean*** of ***100*** minutes and a ***standard deviation*** of ***30*** minutes.

a. A KFUPM student is randomly selected, what is the probability that he studies <u>less than</u> <u>121</u> minutes?



$P\left(x<121\right)=P\left(z<\frac{121-100}{30}\right)$

$=P\left(z<\frac{21}{30}\right)$

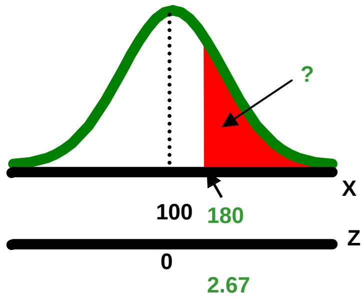
$=P(z<0.70)$ $(1pt)$

$=P(z<0)+P(0<z<0.70)$

$=0.5000+0.2580$ From the std Normal table $(1pt)$

$=0.7580$ $(1pt)$

b. If students who typically obtain A+ grades in their courses study <u>at least</u> 180 minutes daily, what is the percentage of these KFUPM students?



$P\left(x>180\right)=P\left(z>\frac{180-100}{30}\right)$ $(1pt)$

$=P(z>2.66667)\simeq P(z>2.67)$ $(1pt)$

$=1-P(z>2.67)=1-[P(z<0)+P(0<z<2.67)]$

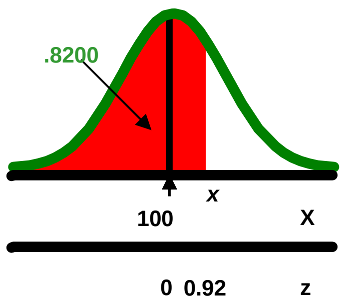
$=1-\left(0.5000+0.4962\right)$ From the std Normal table $(1pt)$

$=1-0.9962$

$=0.0038$ $(1pt)$

0.038% of KFUPM students

c. Find x where 82% of the students study less than x minutes.



$P(X<x)=0.8200$ $(1pt)$

$P(z<z_0)=0.8200$

$P(0<z<z_0)=0.8200-0.5=0.3200$

BUT $z_0=0.92$ from std normal table

$z_0=\frac{x-\mu}{\sigma}=0.92$ $(1pt)$

$x=0.92\left(30\right)+100$ $(1pt)$

$=27.6+100=127.6$ $(1pt)$

d. If 8 KFUPM students are selected at random, then find the probability that at most one of them will study less than 121 minutes. From part a $p=0.7580$

Binomial distribution with $n=8$ $p=0.7580$ $q=1-p=1-0.7580=0.2420$ $(1pt)$

$P\left(x\le 1\right)=P\left(x=0\right)+P(x=1)$ $(1pt)$

$=C_0^8p^0q^8+C_1^8p^1q^7$

$=(1)(1)(0.2420)^8+\frac{8!}{1!7!}(0.7580)(0.2420)^7$ $(1pt)$

$=0.000012+8(0.7580)(0.000049)$

$=0.00012+0.000295$

$=0.000307$ $(1pt)$

Question .5 (*3+2=5-Points*)

The percentage of students who will be admitted to the university after taking an entrance exam is 62%. A random sample of 9 students from those who took the entrance exam is selected. Then:

a. Find the probability that 3 from them will be admitted to the university.

binomial $n=9 \quad p=0.62 \quad q=1-p=0.38 \quad (1pt)$

$$P(x=3)=C_3^9 p^3 q^6$$
$$=\tfrac{9!}{3!6!}(0.62)^3(0.38)^6 \qquad (1pt)$$
$$=\tfrac{9\times8\times7}{3\times2}(0.62)^3(0.38)^6$$
$$=0.060278 \qquad (1pt)$$

b. Find the expected number of students in the sample who will be admitted to the university.

$$E[x]=np=9(0.62) \qquad (1pt)$$
$$=5.58 \qquad (1pt)$$

Question .6(*4+4+4=12-Points*)

Suppose that on the average there are 3 car accidents weekly at the 4th street. Then:

a. Find the probability that there will be at most 2 car accidents at the 4th street next week.

average =3 cars/weekly $=\lambda$ and $t=1$ So, $\lambda t=3(1)=3 \quad (1pt)$ Poisson

$$P(x\le 2)=P(x=0)+P(x=1)+P(x=2) \qquad (1pt)$$
$$=\tfrac{(\lambda t)^0 e^{-\lambda t}}{0!}+\tfrac{(\lambda t)^1 e^{-\lambda t}}{1!}+\tfrac{(\lambda t)^2 e^{-\lambda t}}{2!}$$
$$=e^{-3}+3e^{-3}+\tfrac{3^2}{2}e^{-3} \qquad (1pt)$$
$$=e^{-3}+3e^{-3}+4.5e^{-3}=8.5e^{-3}$$
$$=0.42319 \qquad (1pt)$$

b. Find the probability that there will be at least 1 car accident at the 4th street in the coming 2 weeks.

Average=3 cars/weekly $=\lambda$ and $t=2$ So, $\lambda t=3(2)=6 \quad (1pt)$ Poisson

$$P(x\ge 1)=1-P(x=0)$$
$$=1-(\lambda t)^0\tfrac{e^{-\lambda t}}{0!} \qquad (1pt)$$
$$=1-e^{-6}=1-0.002479 \qquad (1pt)$$
$$=0.997521 \qquad (1pt)$$

c. Find the **mean** and **standard deviation** of the number of car accidents in one year. (Hint: Use one year =53 weeks) $t=1\text{year}=53$ weeks

Mean
$$\mu=E[x]=\lambda t=5(53) \qquad (1pt)$$
$$=159 \qquad (1pt)$$

std deviation
$$\sigma=\sqrt{\lambda t}$$
$$=\sqrt{265} \qquad (1pt)$$
$$=12.6095 \qquad (1pt)$$