Solution: Math 302 – 02   Quiz 4 (A)

Name:..Serial#:..........................

Q.1: Evaluate $\iint\limits_{\Sigma} (\nabla \times \mathbf{F}) \cdot \hat{n}\, d\sigma$, where $\mathbf{F} = \mathbf{xy}\,\mathbf{i} + \mathbf{yz}\,\mathbf{j} + \mathbf{xz}\,\mathbf{k}$, $\sum$ is the part of the plane $2x+4y+z=8$ in the first octant.

$F = [xy, yz, xz]$
$\nabla \times F = [-y, -z, -x]$
$g = 2x + 4y + z - 8$
$N = \nabla g = [2, 4, 1]$
$\|\nabla g\| = \sqrt{21}$
$(\nabla \times F) \cdot N = -2y - 4z - x = -2y - 4(8 - 2x - 4y) - x = 14y - 32 + 7x$
$d\sigma = \sqrt{1 + 2^2 + 4^2} = \sqrt{21} dA$
$\iint\limits_{\Sigma} (\nabla \times F) \cdot \hat{n} d\sigma = \iint\limits_{D} (14y - 32 + 7x)\, dA = \int\limits_0^2 \int\limits_0^{4-2y} (14y - 32 + 7x)\, dx dy = -\frac{160}{3}$

Q.2: Show that for any complex numbers z and w, $|z+w|^2 + |z-w|^2 = 2\left(|z|^2 + |w|^2\right)$

Using $|z|^2 = z\,\bar{z}$
we have $|z+w|^2 + |z-w|^2 = (z+w)(\bar{z}+\bar{w}) + (z-w)(\bar{z}-\bar{w})$
$= z\bar{z} + z\bar{w} + w\bar{z} + w\bar{w} + z\bar{z} - z\bar{w} - w\bar{z} + w\bar{w} = 2z^2 + 2w^2$

Q.3: Transform the equation $|z + 1 + 6i| = |z - 3 + i|$ into rectangular form and write its slope and $y - intercept$.

Let $z = x + iy$, then $|x + 1 + 6i + yi|^2 = x^2 + 2x + 37 + 12y + y^2$
and $|x - 3 + i + yi|^2 = x^2 - 6x + 10 + 2y + y^2$
$|x + 1 + 6i + yi|^2 = |x - 3 + i + yi|^2 \implies 8x + 10y + 27 = 0 \implies y = -\frac{4}{5}x - 2.7$
Slope $m = -\frac{4}{5}$, y-intercept $b = -2.7$
$y = -\frac{4}{5}x - 2.7$