Solution Math 202-091 Quiz 2 (A)

Q.1: Solve the differential equation $(2x+3x^2y^2)\,dx+(2x^3y+\sin y)\,dy=0.$

Sol: $M(x,y)=2x+3x^2y^2$ and $N(x,y)=2x^3y+\sin y$

$M_y=6x^2y=N_x$, the equation is exact.

$$\frac{\partial f}{\partial x}=M(x,y)=2x+3x^2y^2 \Rightarrow f(x,y)=x^2+x^3y^2+g(y)$$
$$\frac{\partial f}{\partial y}=2x^3y+g'(y)=N(x,y)=2x^3y+\sin y \Rightarrow g'(y)=\sin y \Rightarrow g(y)=-\cos y+C$$

Thus one parameter family of solutions is $x^2+x^3y^2-\cos y+C=0.$

Q.2: Solve the initial value problem $\cos x\dfrac{dy}{dx}+(1-\sin x)\,y=\dfrac{1}{1+\sin x},\quad y(0)=1.$

Sol: Given equation can be written as $\dfrac{dy}{dx}+\dfrac{(1-\sin x)}{\cos x}y=\dfrac{1}{\cos x\,(1+\sin x)}$

which is a linear equation with $P(x)=\dfrac{1}{\cos x}-\dfrac{\sin x}{\cos x}=\sec x-\tan x$

$$IF=e^{\int(\sec x-\tan x)dx}=e^{\ln(\sec x+\tan x)+\ln\cos x}=e^{\ln\left(\frac{1+\sin x}{\cos x}\right)\cos x}=1+\sin x$$

$$\frac{d}{dx}\left((1+\sin x)\,y\right)=\sec x \Rightarrow (1+\sin x)y=\ln(\sec x+\tan x)+C$$

$y(0)=1 \Rightarrow 1=\ln(1+0)+C \Rightarrow C=1 \Rightarrow \quad y=\dfrac{\ln(\sec x+\tan x)+1}{1+\sin x}$ is the solution.

Q.3: Transform the equation into a separable equation $(2y^2+3xy)\,dx+x^2dy=0.$

Sol: Given equation is a homogeneous equation.

Putting $y=ux$ and $dy=udx+xdu$, we get $(2x^2u^2+3x^2u)\,dx+x^2\,(udx+xdu)=0$

$(2u^2+3u+u)\,dx+xdu=0 \Rightarrow \dfrac{dx}{x}=\dfrac{-1}{2u^2+4u}du$, a separable equation.

Q.4: Transform the equation into a linear equation $(2xy+y^4)\,dx=5x^2dy.$

Sol: We can write $\dfrac{dy}{dx}=\dfrac{2xy}{5x^2}+\dfrac{y^4}{5x^2} \Rightarrow \dfrac{dy}{dx}-\dfrac{2}{5x}y=\dfrac{1}{5x^2}y^4$, a Bernoulli's equation with $n=4$.

Put $u=y^{1-4}=y^{-3}$ or $y=u^{-\frac{1}{3}} \Rightarrow \dfrac{dy}{dx}=\dfrac{dy}{du}\dfrac{du}{dx}=\dfrac{-1}{3}u^{-\frac{4}{3}}\dfrac{du}{dx}$

$\dfrac{-1}{3}u^{-\frac{4}{3}}\dfrac{du}{dx}-\dfrac{2}{5x}u^{-\frac{1}{3}}=\dfrac{1}{5x^2}u^{-\frac{4}{3}} \Rightarrow \dfrac{du}{dx}+\dfrac{6}{5x}u=\dfrac{-3}{5x^2}$ a linear equation.