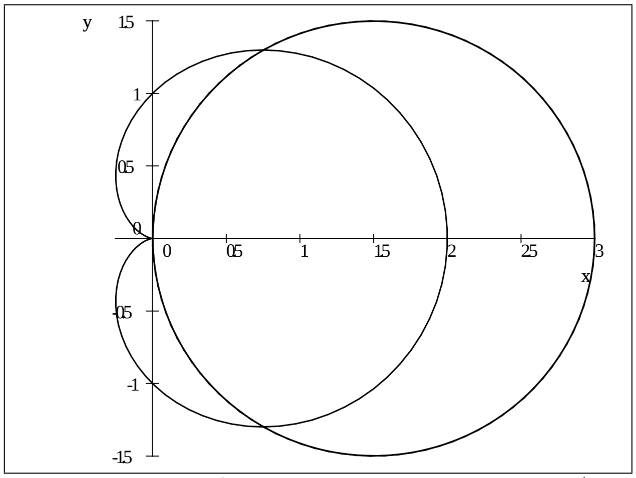
Math 201 Quiz 2 (B)

Q.1: Find area of the region that lies inside the circle $r = 3\sin(\theta)$ and outside the cardioid $r = 1 + \sin(\theta)$.

Sol: The two curves intersect at $\theta = -\frac{\pi}{3}$ and $\theta = \frac{\pi}{3}$. Because of symmetry, we can setup the integral as:

$$A = 2\left(\frac{1}{2}\right)\int_0^{\frac{\pi}{3}} \left(9\cos^2(\theta) - (1+\cos(\theta))^2\right) d\theta = \int_0^{\frac{\pi}{3}} \left(8\cos^2\theta - 2\cos\theta - 1\right) d\theta = \pi.$$



Q.2: Find equation of a sphere that passes through $(-1, 2, 3)$ and has center at $(2, 3, 4)$.

Sol: $r = \sqrt{(2+1)^2 + (3-2)^2 + (4-3)^2} = \sqrt{11}$

$$(x-2)^2 + (y-3)^2 + (z-4)^2 = 11.$$

Q.3: Let $\vec{u} = \langle 2, 1, -1\rangle$ and $\vec{v} = \langle -1, 2, 3\rangle$. Find the following:

(a) $\|\vec{u}+\vec{v}\|$ and $\|\vec{u}-\vec{v}\|$

(b) $\text{Comp}_{(\vec{u}-\vec{v})}(\vec{u}+\vec{v})$

(c) $\text{Proj}_{(\vec{u}+\vec{v})}(\vec{u}-\vec{v})$.

Sol: $\vec{u}+\vec{v} = \langle 1, 3, 2\rangle$ and $\|\vec{u}+\vec{v}\| = \sqrt{1+9+4} = \sqrt{14}$

and $\vec{u}-\vec{v} = \langle 3, -1, -4\rangle$ and $\|\vec{u}-\vec{v}\| = \sqrt{9+1+16} = \sqrt{26}$

$$\text{Comp}_{(\vec{u}-\vec{v})}(\vec{u}+\vec{v}) = \frac{(\vec{u}-\vec{v}).(\vec{u}+\vec{v})}{\|\vec{u}-\vec{v}\|} = \frac{3-3-8}{\sqrt{26}} = \frac{-8}{\sqrt{26}}$$

$$\text{Proj}_{(\vec{u}+\vec{v})}(\vec{u}-\vec{v}) = \frac{(\vec{u}-\vec{v}).(\vec{u}+\vec{v})}{\|\vec{u}+\vec{v}\|}\frac{(\vec{u}+\vec{v})}{\|\vec{u}+\vec{v}\|} = \frac{-8}{14}\langle 1, 3, 2\rangle.$$