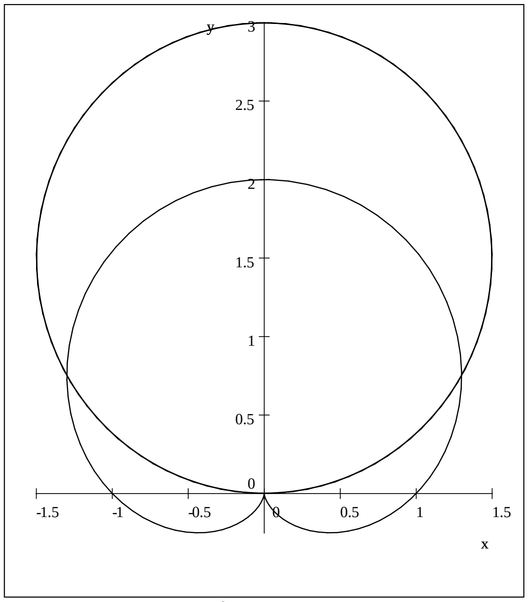
Solution Math 201 Quiz 2 (A)

Q.1: Find area of the region that lies inside the circle $r = 3\sin(\theta)$ and outside the cardioid $r = 1 + \sin(\theta)$.

Sol: The two curves intersect at $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$. Because of symmetry, we can setup the integral as:

$$A = 2\left(\frac{1}{2}\right)\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(9\sin^2(\theta) - (1+\sin(\theta))^2\right) d\theta = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (3 - 4\cos 2\theta - 2\sin\theta)\, d\theta = \pi$$



Q.2: Find equation of a sphere that passes through $(2,3,4)$ and has center at $(-1,2,3)$.

Sol: $r = \sqrt{(2+1)^2 + (3-2)^2 + (4-3)^2} = \sqrt{11}$

$(x+1)^2 + (y-2)^2 + (z-3)^2 = 11.$

Q.3: Let $\vec{u} = \langle -2, 2, 1\rangle$ and $\vec{v} = \langle 1, 2, -3\rangle$. Find the following:

(a) $\|\vec{u}+\vec{v}\|$ and $\|\vec{u}-\vec{v}\|$

(b) $\text{Comp}_{(\vec{u}+\vec{v})}(\vec{u}-\vec{v})$

(c) $\text{Proj}_{(\vec{u}-\vec{v})}(\vec{u}+\vec{v})$.

Sol: $\vec{u}+\vec{v} = \langle -1, 4, -2\rangle$ and $\|\vec{u}+\vec{v}\| = \sqrt{1+16+4} = \sqrt{21}$

and $\vec{u}-\vec{v} = \langle -3, 0, 4\rangle$ and $\|\vec{u}-\vec{v}\| = \sqrt{9+0+16} = 5$

$$\text{Comp}_{(\vec{u}+\vec{v})}(\vec{u}-\vec{v}) = \frac{(\vec{u}-\vec{v})\,.\,(\vec{u}+\vec{v})}{\|\vec{u}+\vec{v}\|} = \frac{3+0-8}{\sqrt{21}} = \frac{-5}{\sqrt{21}}$$

$$\text{Proj}_{(\vec{u}-\vec{v})}(\vec{u}+\vec{v}) = \frac{(\vec{u}-\vec{v})\,.\,(\vec{u}+\vec{v})}{\|\vec{u}-\vec{v}\|}\,\frac{(\vec{u}-\vec{v})}{\|\vec{u}-\vec{v}\|} = \frac{-5}{25}\langle -3, 0, 4\rangle.$$