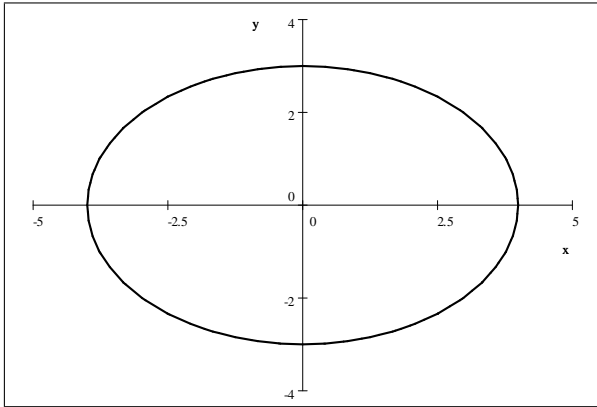


Name:.....ID#:.....

**Q.1:** Eliminate the parameter  $t$  from the parametric equations  $x = 4 \cos(t)$ ,  $y = 3 \sin(t)$  to find a cartesian equation. Sketch the graph and mark the direction in which the curve is traced.

Sol:  $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$  or  $\frac{x^2}{16} + \frac{y^2}{9} = 1$  is an ellipse traced anti clockwise starting at  $(4,0)$ , since

$t$	0	$\frac{\pi}{2}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
$x$	4	0	-4	0	4
$y$	0	3	0	-3	0



**Q.2:** Find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  for the parametric equations given in Question 1. Also find equation of the tangent line to the curve at  $t = \frac{\pi}{4}$ .

Sol:  $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3 \cot t}{-4 \sin t} = -\frac{3}{4} \cot t$ , and  $\frac{d^2y}{dx^2} = \frac{\frac{d}{dt} \left( \frac{dy}{dx} \right)}{\frac{dx}{dt}} = \frac{-\frac{3}{4} \csc^2 t}{-4 \sin t} = \frac{-3}{16 \sin^3 t}$ .

Slope of the tangent at  $t = \frac{\pi}{4}$ , is  $m = -\frac{3}{4} \cot \left( \frac{\pi}{4} \right) = -\frac{3}{4}$  and points is  $x = 4 \cos \left( \frac{\pi}{4} \right) = 4 \frac{\sqrt{2}}{2}$ ,  $y = 3 \sin \left( \frac{\pi}{4} \right) = 3 \frac{\sqrt{2}}{2}$ . Equation of tangent line is  $y - \frac{3\sqrt{2}}{2} = -\frac{3}{4} \left( x - \frac{4\sqrt{2}}{2} \right)$ .

**Q.3:** Sketch the graph of the polar equation  $r = 3 - 3 \sin(\theta)$ .

