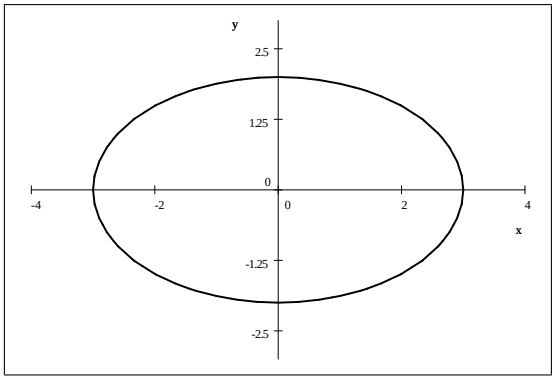
Solution Math 201 Quiz 1 (A)

Q.1: Elliminate the parameter t from the parametric equations $x = 3\sin(t),\ y = 2\cos(t)$ to find a cartesian equation. Sketch the graph and mark the direction in which the curve is traced.

Sol: $\left(\frac{x}{3}\right)^2+\left(\frac{y}{2}\right)^2 = 1$ or $\frac{x^2}{9}+\frac{y^2}{4} = 1$ is an ellipse traced clockwise starting at $(0,2)$, since

t	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
x	0	3	0	-3	0
y	2	0	-2	0	2



Q.2: Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for the parametric equations given in Question 1. Also find equation of the tangent line to the curve at $t = \frac{\pi}{4}$.

Sol: $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2\sin t}{3\cos t} = -\frac{2}{3}\tan t$, and $\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{-\frac{2}{3}\sec^2 t}{3\cos t} = \frac{-2}{9\cos^3 t}$.

Slope of the tangent at $t = \frac{\pi}{4}$, is $m = -\frac{2}{3}\tan\left(\frac{\pi}{4}\right) = -\frac{2}{3}$ and points is $x = 3\sin\left(\frac{\pi}{4}\right) = 3\frac{\sqrt{2}}{2}$, $y = 2\cos\left(\frac{\pi}{4}\right) = 2\frac{\sqrt{2}}{2}$. Equation of tangent line is $y - \sqrt{2} = -\frac{2}{3}\left(x - \frac{3\sqrt{2}}{2}\right)$.

Q.3: Sketch the graph of the polar equation $r = 2 + 2\cos(\theta)$.