King Fahd University of Petroleum and Minerals

Department of Mathematical Sciences

Exam # 1 for Math 302 – 03 (A)

Name:... .....Serial #:..........................

Note: Show all your work to get full credit.

Q.1: Let the set S consists of all vectors in R^5 whose last components is twice the first component, second component is sum of third and fourth component. Show that whether the set S is a subspace of R^5 or not.

Q.2: Let S consists of all vectors in R^5 whose third component is 1. Determine whether S is a subspace of R^5 or not.

Q.3: Let $F_1,\ F_2,\ ..., F_n$ are nonzero mutually orthogonal vectors in R^n. Show that $F_1,\ F_2,\ ..., F_n$ are linearly independent.

Q.4: Let S consists of all vectors $(x, y, x+y, x-y, z)$ in R^5. Determine a basis for S and dimension of the subspace S.

Q.5: Find the general solution of the system

$$\begin{aligned} 3x + 6y + 9z &= 0 \\ 4x + y - 2z &= 0 \\ 4x - 4y + 6z &= 0 \end{aligned}$$

Q.6: Let λ be an eigenvalue of A with eigenvector X and μ be an eigenvalue of A with eigenvector Y. If $\lambda \neq \mu$, show that X and Y are linearly independent.

Q.7: Consider the system

$$\begin{aligned} x - y + 2z &= 3 \\ -4x + y + 7z &= -5 \\ -2x - y + 11z &= 14 \end{aligned}$$

Find rank of the coefficient matrix and the augmented matrix and determine if the system has a solution or there is no solution.

Q.8: Show that all the eigenvalues of a 2×2 symmetric matrix with real entries are real. Can a nonreal matrix have real eigenvalues? If yes, give an example.

Q.9: Let $A = \begin{bmatrix} -1 & 0 \\ 1 & -5 \end{bmatrix}$. Find A^{10}.

Q.10: Let $\lambda_1 = 2$, $\lambda_2 = 1+\sqrt{3}$, and $\lambda_3 = 1-\sqrt{3}$ are eigenvalues of $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$. Find eigenvector corresponding to these eigenvalues and show that they are orthogonal. Also find the corresponding orthonormal eigenvectors.