Math 241 – Quiz # 2c

Name: Solution

Sr #: ______________

1. Let $B = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and $C = \begin{bmatrix} -g & -h & -i \\ 7d & 7e & 7f \\ a & b & c \end{bmatrix}$. If det($B$) = -3, find det($C$) ?

$$\det(C) = \begin{vmatrix} -g & -h & -i \\ 7d & 7e & 7f \\ a & b & c \end{vmatrix} = 7\begin{vmatrix} -g & -h & -i \\ d & e & f \\ a & b & c \end{vmatrix} \xrightarrow{-R_1} = 7(-1)\begin{vmatrix} g & h & i \\ d & e & f \\ a & b & c \end{vmatrix} \xrightarrow{R_1 \leftrightarrow R_3}$$

$$= 7(-1)(-1)\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 7(-1)(-1)\det(B)$$

$$= 7(-1)(-1)(-3)$$

$$= -21$$

2. Find the values of k for which the matrix $B = \begin{bmatrix} 2 & -2 & 0 \\ 1 & 3 & 1 \\ 0 & 2 & K \end{bmatrix}$ is invertible.

B is invertible iff $\det(B) \neq 0$.

$$\det(B) = \begin{vmatrix} 2 & -2 & 0 \\ 1 & 3 & 1 \\ 0 & 2 & k \end{vmatrix} = 2\begin{vmatrix} 3 & 1 \\ 2 & k \end{vmatrix} + 2\begin{vmatrix} 1 & 1 \\ 0 & k \end{vmatrix}$$

$$= 2(3k-2) + 2k$$

$$= 6k - 4 + 2k$$

$$= 8k - 4$$

B is invertible $\iff 8k - 4 \neq 0$

$\iff k \neq \frac{1}{2}$

So, B is invertible for all values of k except $k = \frac{1}{2}$.

3. Consider the following system of linear equations:

$$-2x_1 + 3x_2 - x_3 = 1$$
$$x_1 + 2x_2 - x_3 = 4$$
$$-2x_1 - x_2 + x_3 = -3$$

Use Cramer's rule to solve for x_1?

The coefficients matrix $A = \begin{bmatrix} -2 & 3 & -1 \\ 1 & 2 & -1 \\ -2 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 4 \\ -3 \end{bmatrix}$

$$|A| = \begin{vmatrix} -2 & 3 & -1 \\ 1 & 2 & -1 \\ -2 & -1 & 1 \end{vmatrix} = -2\begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix} - 3\begin{vmatrix} 1 & -1 \\ -2 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 2 \\ -2 & -1 \end{vmatrix}$$
$$= -2(1) - 3(-1) - 1(3) = -2 + 3 - 3 = \boxed{-2}$$

$$|A_1| = \begin{vmatrix} 1 & 3 & -1 \\ 4 & 2 & -1 \\ -3 & -1 & 1 \end{vmatrix} = 1\begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix} - 3\begin{vmatrix} 4 & -1 \\ -3 & 1 \end{vmatrix} - 1\begin{vmatrix} 4 & 2 \\ -3 & -1 \end{vmatrix}$$
$$= 1 - 3(1) - 2 = \boxed{-4}$$

$$x_1 = \frac{|A_1|}{|A|} = \frac{-4}{-2} = 2$$