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$\boxed{4}$ The set of all pairs $(0,y) \in \mathbb{R}^2$ with the standard operations is a vector space. [See example 2 in your notes].

$\boxed{5}$ The set of all pairs $(x,y) \in \mathbb{R}^2$ with $x \geq 0$, with the standard operations is NOT a vector space, since Axiom 6 is not satisfied:

- Take $u = (3,2)$, $k = -1$. Then $ku = -1(3,2) = (-3,-2) \notin V$.
- Also, Axiom 5 is not satisfied:
  Take $u = (3,2)$. Then $\nexists\, v \in V$ such that $u + v = (0,0) = 0$
  i.e. $-u$ does not exist.

$\boxed{7}$ The set of all triples $(x,y,z) \in \mathbb{R}^3$ with the standard vector addition but with scalar multiplication defined as:

$$k(x,y,z) = (k^2x, k^2y, k^2z)$$

is not a vector space since Axiom 8 is not satisfied:

For instance, take $u = (1,2,1)$, $k = 2$, $m = 3$. Then

$$ku = 2(1,2,1) = (4,8,4) \quad , \quad mu = 3(1,2,1) = (9,18,9)$$

$$(k+m)u = (2+3)(1,2,1) = (25,50,25)$$

$$ku + mu = (13,26,13)$$

$$\Rightarrow (k+m)u \neq ku + mu.$$

$\boxed{9}$ $V = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} : a, b \in \mathbb{R} \right\}$ is a vector space.

$\boxed{11}$ $V = \{ (x,1) : x \in \mathbb{R} \}$ withe the operations:

$(x,1) + (x',1) = (x+x',1)$ and $k(x,1) = (k^2 x, 1)$.

V is Not a vector space.

<u>Note that</u> the addition is closed:

Since $u, v \in V \Rightarrow u+v \in V$

<u>Also,</u> The scalar multiplication is closed:

Since $u \in V$ and k is scalar $\Rightarrow ku \in V$.

However, V is Not a vector space Since

Axiom 8 is not satisfied (see exercise 7 above).

You have to show that $(k+m)u \neq ku + mu$.