HW Page 127

$\boxed{4}$ To verify: $\det(kA) = k^n \det(A)$.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 1 & -2 \end{bmatrix} ; \quad k = 3$$

We need to verify that $\det(3A) = 3^3 \det(A)$

Now, $\det(A) = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 1 & -2 \end{vmatrix} = 1 \begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} = -7$

$$3A = \begin{bmatrix} 3 & 3 & 3 \\ 0 & 6 & 9 \\ 0 & 3 & -6 \end{bmatrix} \Rightarrow \det(3A) = \begin{vmatrix} 3 & 3 & 3 \\ 0 & 6 & 9 \\ 0 & 3 & -6 \end{vmatrix} = 3 \begin{vmatrix} 6 & 9 \\ 3 & -9 \end{vmatrix}$$

$$= -189$$

$$3^3 \det(A) = 27 \det(A) = 27(-7) = -189 = \det(3A)$$

Hence $\det(3A) = 3^3 \det(A)$.

$\boxed{5}$ Just calculate $\det(A)$, $\det(B)$, $\det(AB)$, $\det(A+B)$

$\boxed{7}$ $A = \begin{bmatrix} 3 & 6 & 1 \\ 0 & 2 & -4 \\ 0 & 0 & 1 \end{bmatrix}$

$$\det(A) = \begin{vmatrix} 3 & 6 & 1 \\ 0 & 2 & -4 \\ 0 & 0 & 1 \end{vmatrix} = 3 \begin{vmatrix} 2 & -4 \\ 0 & 1 \end{vmatrix} = 6 \neq 0$$

$\therefore \det(A) \neq 0 \Rightarrow A$ is invertible.

10 $A = \begin{bmatrix} -3 & 0 & 1 \\ 5 & 0 & 6 \\ 8 & 0 & 3 \end{bmatrix}$. $\det(A) = 0 \Rightarrow A$ is not invertible.

17 $A = \begin{bmatrix} 1 & 3 & k \\ 2 & 1 & 3 \\ 4 & 6 & 2 \end{bmatrix}$

$$\det(A) = \begin{vmatrix} 1 & 3 & k \\ 2 & 1 & 3 \\ 4 & 6 & 2 \end{vmatrix} = 1\begin{vmatrix} 1 & 3 \\ 6 & 2 \end{vmatrix} - 3\begin{vmatrix} 2 & 3 \\ 4 & 2 \end{vmatrix} + k\begin{vmatrix} 2 & 1 \\ 4 & 6 \end{vmatrix}$$

$$= -16 - 3(-8) + 8k = 8 + 8k$$

A is invertible when $\det(A) \neq 0$

$\Rightarrow \quad 8 + 8k \neq 0$

$\Rightarrow \quad k \neq -1$

So, A is invertible for all values of k except $k = -1$.

23 Use $A^{-1} = \dfrac{1}{\det(A)} \operatorname{adj}(A)$ to get

$A^{-1} = \begin{bmatrix} -4 & 3 & 0 & -1 \\ 2 & -1 & 0 & 0 \\ -7 & 0 & -1 & 8 \\ 6 & 0 & 1 & -7 \end{bmatrix}$. OR use row reduction to get A^{-1}.

$\boxed{27}$

$$\begin{aligned} x_1 - 3x_2 + x_3 &= 4 \\ 2x_1 - x_2 \phantom{{}+x_3} &= -2 \\ 4x_1 \phantom{{}-3x_2} - 3x_3 &= 0 \end{aligned}$$

To use <u>Cramer's rule</u> to solve the system:

$$A = \begin{bmatrix} 1 & -3 & 1 \\ 2 & -1 & 0 \\ 4 & 0 & -3 \end{bmatrix}, \qquad B = \begin{bmatrix} 4 \\ -2 \\ 0 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -3 & 1 \\ 2 & -1 & 0 \\ 4 & 0 & -3 \end{vmatrix} = 1\begin{vmatrix} 2 & -1 \\ 4 & 0 \end{vmatrix} - 3\begin{vmatrix} 1 & -3 \\ 2 & -1 \end{vmatrix} = 4 - 15 = -11$$

$$|A_1| = \begin{vmatrix} 4 & -3 & 1 \\ -2 & -1 & 0 \\ 0 & 0 & -3 \end{vmatrix} = -3\begin{vmatrix} 4 & -3 \\ -2 & -1 \end{vmatrix} = -3(-10) = 30$$

$$|A_2| = \begin{vmatrix} 1 & 4 & 1 \\ 2 & -2 & 0 \\ 4 & 0 & -3 \end{vmatrix} = 1\begin{vmatrix} 2 & -2 \\ 4 & 0 \end{vmatrix} - 3\begin{vmatrix} 1 & 4 \\ 2 & -2 \end{vmatrix} = 8 - 3(-10) = 38$$

$$|A_3| = \begin{vmatrix} 1 & -3 & 4 \\ 2 & -1 & -2 \\ 4 & 0 & 0 \end{vmatrix} = 4\begin{vmatrix} -3 & 4 \\ -1 & -2 \end{vmatrix} = 4(10) = 40$$

$$x_1 = \frac{|A_1|}{|A|} = -\frac{30}{11}$$

$$x_2 = \frac{|A_2|}{|A|} = -\frac{38}{11}$$

$$x_3 = \frac{|A_3|}{|A|} = -\frac{40}{11}$$

$\boxed{29}$

$$3x_1 - x_2 + x_3 = 4$$
$$-x_1 + 7x_2 - 2x_3 = 1$$
$$2x_1 + 6x_2 - x_3 = 5$$

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 7 & -2 \\ 2 & 6 & -1 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 3 & -1 & 1 \\ -1 & 7 & -2 \\ 2 & 6 & -1 \end{vmatrix} = 0$$

$\Rightarrow$ Cramer's Rule does not apply.

$\boxed{31}$ $y = 0$

$\boxed{34}$ A is 4×4. $\det(A) = -2$

(a) $\det(-A) = -2$ why?

(b) $\det(A^{-1}) = -\frac{1}{2}$ since $\det(A^{-1}) = \frac{1}{\det(A)}$

(c) $\det(2A^T) = 2^4 \det(A^T) = 2^4 \det(A) = 16(-2) = -32$

(d) $\det(A^3) = \det(A)\det(A)\det(A) = (-2)^3 = -8$