Math 202 – Quiz # 4b

Name: Solution   Ser. #______

Verify that the DE is Bernoulli and then solve the IVP:

$$tx\frac{dx}{dt} = 3x^2 + t^2 \,, \qquad x(-1) = 2$$

$$\frac{dx}{dt} = \frac{3}{t}x + \frac{t}{x}$$

$(*)$ $\dfrac{dx}{dt} - \dfrac{3}{t}x = t x^{-1}$ which is Bernoulli of the form:

$$\frac{dx}{dt} + p(t)x = f(t)x^n$$

Let $w = x^{1-n} = x^{1-(-1)} = x^2 \Rightarrow x = w^{\frac{1}{2}}$, $x^{-1} = w^{-\frac{1}{2}}$ $\Rightarrow \dfrac{dx}{dt} = \dfrac{1}{2}w^{-\frac{1}{2}}\dfrac{dw}{dt}$

substitute in $(*)$:

$$\frac{1}{2}w^{-\frac{1}{2}}\frac{dw}{dt} - \frac{3}{t}w^{\frac{1}{2}} = t w^{-\frac{1}{2}}$$

$(**)$ $\dfrac{dw}{dt} - \dfrac{6}{t}w = 2t$ ← [linear in w]

$$\mu = e^{-\int \frac{6}{t}dt} = e^{-6\ln t} = t^{-6}$$

Multiply both sides of $(**)$ by t^{-6}:

$$\frac{d}{dt}\left[w t^{-6}\right] = 2t^{-5}$$

$$w t^{-6} = \int 2t^{-5}dt = \frac{-1}{2}t^{-4} + c$$

$$w = -\frac{1}{2}t^2 + ct^6$$

$$x^2 = -\frac{t^2}{2} + ct^6$$

using the condition $x(-1) = 2 \Rightarrow 4 = -\frac{1}{2} + c \Rightarrow \boxed{c = \frac{9}{2}}$

$\therefore$ the solution is:

$$x^2 = -\frac{t^2}{2} + \frac{9}{2}t^6$$

i.e. $2x^2 = 9t^6 - t^2$