Math 202 – Quiz # 4

Name: Solution Ser. #______

Verify that the following DE is homogeneous and then solve it:

$$\underbrace{(xy + x^2 + y^2)}_{M}dx - \underbrace{x^2}_{N}dy = 0$$

$M(tx,ty) = txty + (tx)^2 + (ty)^2 = t^2xy + t^2x^2 + t^2y^2 = t^2(xy + x^2 + y^2) = t^2M(x,y)$

$N(tx,ty) = (tx)^2 = t^2x^2 = t^2N(x,y)$. $\therefore$ The given DE is homogeneous

Put $y = ux$

$dy = udx + xdu$

substitute in the given DE:

$$(x^2u + x^2u^2 + x^2)dx - x^2(udx + xdu) = 0$$

$$(x^2u + x^2u^2 + x^2 - x^2u)dx - x^3du = 0$$

$$x^2(u^2+1)dx - x^3du = 0$$

$$\frac{dx}{x} - \frac{du}{u^2+1} = 0$$

$$\frac{du}{u^2+1} = \frac{dx}{x}$$

$$\tan^{-1}u = \ln|x| + c$$

$$u = \tan(\ln|x| + c)$$

$$\frac{y}{x} = \tan(\ln|x| + c)$$

$\therefore$ The solution is

$$y = x\tan(\ln|x| + c)$$