Math 202 – Quiz # 3b

Name: Solution Ser. #_______

Convert the following differential equation into an exact equation, and then solve it:

$$\underset{M}{xy}\,dx + \underset{N}{(2x^2 + 3y^2 - 20)}\,dy = 0$$

$M = xy$, $N = 2x^2+3y^2-20$

$\frac{\partial M}{\partial y} = x$, $\frac{\partial N}{\partial x} = 4x \Rightarrow \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \Rightarrow$ Not exact

$\frac{1}{N}\left[\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right] = \frac{1}{2x^2+3y^2-20}\left[x-4x\right] = \frac{-3x}{2x^2+3y^2-20} = q(x,y)$!!

a function of both x & y.

$\frac{1}{M}\left[\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right] = \frac{1}{xy}\left[-3x\right] = \frac{-3}{y} = g(y)$ ✓

$\therefore$ An integrating factor is $\mu = e^{-\int g(y)dy} = e^{-\int \frac{-3}{y}dy} = e^{\int \frac{3}{y}dy} = e^{3\ln y} = y^3$

Multiply both sides of the given DE by y^3 to gen an Exact equation:

$$y^3\, xy\,dx + y^3(2x^2+3y^2-20)\,dy = 0$$

$$xy^4\,dx + (2x^2y^3+3y^5-20y^3)\,dy = 0$$

(Exact) $M = xy^4$, $N = 2x^2y^3+3y^5-20y^3$

$\Rightarrow \exists f$ such that $\frac{\partial f}{\partial x} = M = xy^4$

$\frac{\partial f}{\partial y} = N = 2x^2y^3+3y^5-20y^3$

$f(x,y) = \int xy^4\,dx = \frac{1}{2}x^2y^4 + g(y)$

$\frac{\partial f}{\partial y} = 2x^2y^3 + g'(y) = N = 2x^2y^3+3y^5-20y^3$

$\Rightarrow g'(y) = 3y^5 - 20y^3$

$g(y) = \int(3y^5-20y^3)\,dy = \frac{1}{2}y^6 - 5y^4 + C_1$

$\therefore f(x,y) = \frac{1}{2}x^2y^4 + \frac{1}{2}y^6 - 5y^4 + C_1$

The solution is: $\frac{1}{2}x^2y^4 + \frac{1}{2}y^6 - 5y^4 = C$.