Math 202 – Quiz # 3

Name: Solution Ser. #______

Solve the following IVP: $\underbrace{(2xy-\sec^2 x)}_{M}dx+\underbrace{(x^2+2y)}_{N}dy=0$, $y(\pi)=1$

$$M = 2xy - \sec^2 x \quad , \quad N = x^2 + 2y$$

$$\frac{\partial M}{\partial y} = 2x \quad , \quad \frac{\partial N}{\partial x} = 2x$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow \text{Exact equation} \Rightarrow \exists f \text{ such that}$$

$$\frac{\partial f}{\partial x} = M = 2xy - \sec^2 x \ \& \ \frac{\partial f}{\partial y} = N = x^2 + 2y$$

$$f(x,y) = \int (2xy - \sec^2 x)\,dx$$

$$= x^2 y - \tan x + g(y)$$

$$\frac{\partial f}{\partial y} = x^2 + g'(y) = N = x^2 + 2y$$

$$\Rightarrow g'(y) = 2y$$

$$\Rightarrow g(y) = y^2 + c_1$$

$$\therefore f(x,y) = x^2 y - \tan x + y^2 + c_1$$

The general solution is: $x^2 y - \tan x + y^2 = c$

Using the condition $y(\pi)=1$, we have $\pi^2 + 1 = c$

$\therefore$ the solution of the given IVP is:

$$x^2 y - \tan x + y^2 = \pi^2 + 1$$