Math 202 – Quiz # 8b

Name: Solution   Sr #: ____________

Consider the matrix $A=\begin{bmatrix}-1 & 1 & 0\\ 1 & 2 & 1\\ 0 & 3 & -1\end{bmatrix}$.

(1) Write the characteristic polynomial of A.

(2) Find the eigen values of A.

(3) Find the eigen vectors of A which correspond to the eigen values in (2).

(1) $|A-\lambda I|=0 \Rightarrow \begin{vmatrix}-1-\lambda & 1 & 0\\ 1 & 2-\lambda & 1\\ 0 & 3 & -1-\lambda\end{vmatrix}=0 \Rightarrow (-1-\lambda)\left[(2-\lambda)(-1-\lambda)-3\right]-(-1-\lambda)=0$ ---(*)

$\Rightarrow \boxed{-\lambda^3+7\lambda+6 = P(\lambda)}$ is the char. poly.

(2) From (*) $\Rightarrow -(1+\lambda)\left[\lambda^2-\lambda-6\right]=0 \Rightarrow -(1+\lambda)(\lambda-3)(\lambda+2)=0$

$\Rightarrow \boxed{\lambda=-1,\ 3,\ -2}$ are the eigen values.

(3) For $\lambda=-1$: $(A+I)K=0 \Rightarrow \begin{bmatrix}0 & 1 & 0\\ 1 & 3 & 1\\ 0 & 3 & 0\end{bmatrix}\begin{bmatrix}k_1\\ k_2\\ k_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

$\left[\begin{array}{ccc|c}0 & 1 & 0 & 0\\ 1 & 3 & 1 & 0\\ 0 & 3 & 0 & 0\end{array}\right]\longrightarrow\left[\begin{array}{ccc|c}1 & 3 & 1 & 0\\ 0 & 1 & 0 & 0\\ 0 & 3 & 0 & 0\end{array}\right]\longrightarrow\left[\begin{array}{ccc|c}1 & 3 & 1 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\end{array}\right] \Rightarrow \begin{cases}k_2=0\\ k_1=-k_3\end{cases} \Rightarrow \boxed{K_1=\begin{bmatrix}-1\\0\\1\end{bmatrix}}$ is the eigenvector ass. with $\lambda=-1$

For $\lambda=-2$: $(A+2I)K=0 \Rightarrow \left[\begin{array}{ccc|c}1 & 1 & 0 & 0\\ 1 & 4 & 1 & 0\\ 0 & 3 & 1 & 0\end{array}\right]\Rightarrow\left[\begin{array}{ccc|c}1 & 1 & 0 & 0\\ 0 & 3 & 1 & 0\\ 0 & 3 & 1 & 0\end{array}\right]\rightarrow\left[\begin{array}{ccc|c}1 & 1 & 0 & 0\\ 0 & 3 & 1 & 0\\ 0 & 0 & 0 & 0\end{array}\right]\Rightarrow\begin{cases}k_2=-\frac{k_3}{3}\\ k_1=-k_2\end{cases}$

$\Rightarrow \boxed{K_2=\begin{bmatrix}-1\\0\\1\end{bmatrix}}$ is the eigen vector ass. with $\lambda=-2$

For $\lambda=3$: $(A-3I)K=0 \Rightarrow \left[\begin{array}{ccc|c}-4 & 1 & 0 & 0\\ 1 & -1 & 1 & 0\\ 0 & 3 & -4 & 0\end{array}\right]$

$\longrightarrow\left[\begin{array}{ccc|c}1 & -1 & 1 & 0\\ -4 & 1 & 0 & 0\\ 0 & 3 & -4 & 0\end{array}\right]\longrightarrow\left[\begin{array}{ccc|c}1 & -1 & 1 & 0\\ 0 & -3 & 4 & 0\\ 0 & 3 & -4 & 0\end{array}\right]\longrightarrow\left[\begin{array}{ccc|c}1 & -1 & 1 & 0\\ 0 & -3 & 4 & 0\\ 0 & 0 & 0 & 0\end{array}\right]\Rightarrow\begin{cases}k_2=\frac{4}{3}k_3\\ k_1=k_2-k_3\end{cases}$

$\Rightarrow \boxed{K_3=\begin{bmatrix}1\\4\\3\end{bmatrix}}$ is the eigen vector ass. with $\lambda=3$