

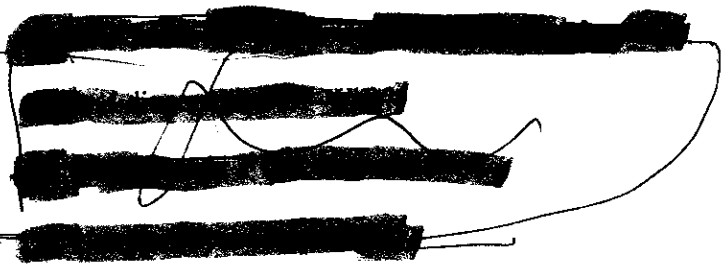
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Math 260 - Quiz #

Name: Solution

Sr #: _____

Consider the matrix $A = \begin{bmatrix} 3 & -1 & 0 \\ 0 & 2 & 0 \\ -1 & 1 & 2 \end{bmatrix}$.

- (1) Write the characteristic polynomial of A .
- (2) Find the eigen values of A .
- (3) Find the eigen vectors of A which correspond to the eigen values in (2).



$$\begin{aligned}
 (1) \quad |A - \lambda I| &= \begin{vmatrix} 3-\lambda & -1 & 0 \\ 0 & 2-\lambda & 0 \\ -1 & 1 & 2-\lambda \end{vmatrix} = (2-\lambda)(3-\lambda)(2-\lambda) \\
 &= (3-\lambda)(2-\lambda)^2 \\
 &= -\lambda^3 + 5\lambda^2 - 10\lambda + 12
 \end{aligned}$$

The characteristic polynomial is $P(\lambda) = -\lambda^3 - 5\lambda^2 - 10\lambda + 12$

(2) From above, $|A - \lambda I| = (3-\lambda)(2-\lambda)^2 = 0$

\therefore the eigen values of A are: 3, 2, 2

(3) To find the eigenvectors:

For $\lambda=3$

$$(A-3I)X=0 \Rightarrow \begin{bmatrix} 0 & -1 & 0 \\ 0 & -1 & 0 \\ -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\Rightarrow x_2=0, x_1=-x_3. \text{ Take } x_3=-t \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} t \\ 0 \\ -t \end{bmatrix} = t \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}.$$

$\therefore E_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ is an eigen vector corresponds to $\lambda=3$

For $\lambda=2$

$$(A-2I)X=0 \Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\Rightarrow x_1 = x_2 = t, x_3 = s \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} t \\ t \\ s \end{bmatrix} = t \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

\therefore We have two eigen vectors E_2, E_3 correspond to $\lambda=2$,

where $E_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, E_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$