Math 260 – Quiz # 8b

Name: Solution   Sr #: ____________

Consider the matrix $A = \begin{bmatrix} 3 & -1 & 0 \\ 0 & 2 & 0 \\ -1 & 1 & 2 \end{bmatrix}$.

2 **(1)** Write the characteristic polynomial of A.

1 **(2)** Find the eigen values of A.

2 **(3)** Find the eigen vectors of A which correspond to the eigen values in (2).

[redacted]

(1) $$|A-\lambda I| = \begin{vmatrix} 3-\lambda & -1 & 0 \\ 0 & 2-\lambda & 0 \\ -1 & 1 & 2-\lambda \end{vmatrix} = (2-\lambda)(3-\lambda)(2-\lambda)$$
$$= (3-\lambda)(2-\lambda)^2$$
$$= -\lambda^3 + 5\lambda^2 - 10\lambda + 12$$

The characteristic polynomial is $P(\lambda) = -\lambda^3 - 5\lambda^2 - 10\lambda + 12$

(2) From above, $|A-\lambda I| = (3-\lambda)(2-\lambda)^2 = 0$

$\therefore$ the eigen values of A are: $3, 2, 2$.

(3) <u>To find the eigenvectors:</u>

<u>For $\lambda = 3$</u>

$$(A-3I)X = 0 \Rightarrow \begin{bmatrix} 0 & -1 & 0 \\ 0 & -1 & 0 \\ -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\Rightarrow x_2 = 0, \quad x_1 = -x_3$. Take $x_3 = -t \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} t \\ 0 \\ -t \end{bmatrix} = t\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$.

$\therefore E_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ is an eigen vector corresponds to $\lambda = 3$

<u>For $\lambda = 2$</u>

$$(A-2I)\bar{X} = 0 \Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\Rightarrow x_1 = x_2 = t, \ x_3 = s \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} t \\ t \\ s \end{bmatrix} = t\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + s\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\therefore$ we have two eigen vectors E_2, E_3 correspond to $\lambda = 2$,

where $E_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad E_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.