### 6.1.1<sub>1</sub>

#### **Section 6.1** *Solution about ordinary points*

#### **6.1.1 Review of power series**

Here we will briefly review those results of powers series which we need to understand method of series solutions of differential equations. In case you have forgotten the stuff related to power series, my advice is to consult your Calculus notes to refresh your understanding of power series.

### **Learning Outcomes**

After completing this sub-section, you will inshaAllah be able to

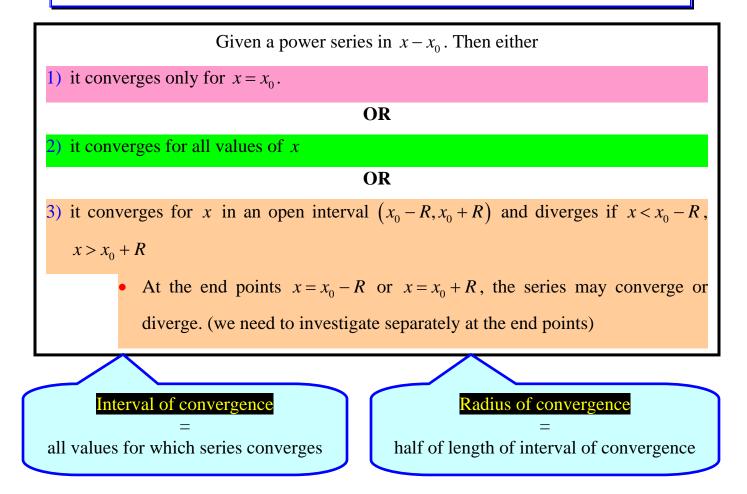
- 1. recall what is meant by power series
- 2. recall what is meant by interval & radius of convergence of a power series
- 3. recall how to find interval & radius of convergence of a power series
- 4. handle problems related to shifting the index of summation
- 5. understand what is meant by analytic points
- 6. recall how to write Taylor series of a function about analytic points

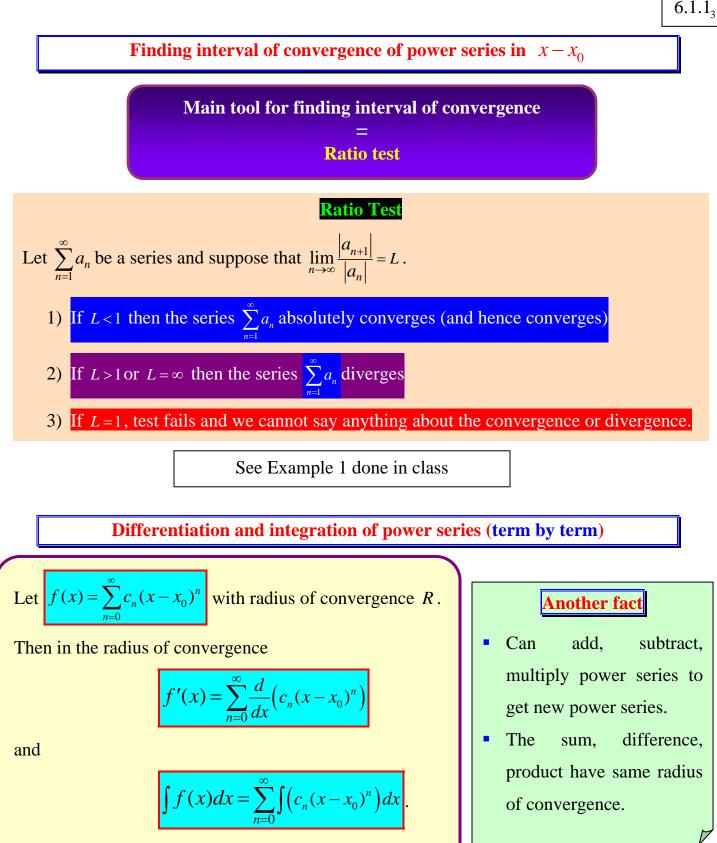
# **Power series in** $x - x_0$

A power series about the point  $x_0$  is series in the powers of  $x - x_0$ , of the form

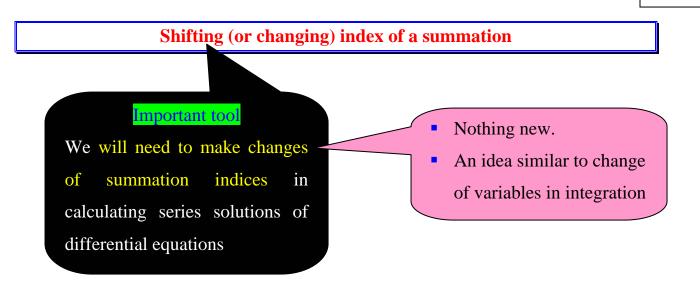
$$\sum_{n=0}^{\infty} c_n (x-x_0)^n = c_0 + c_1 (x-x_0) + c_2 (x-x_0)^2 + \dots + c_n (x-x_0)^n + \dots$$

### **Convergence of power series in** $x - x_0$



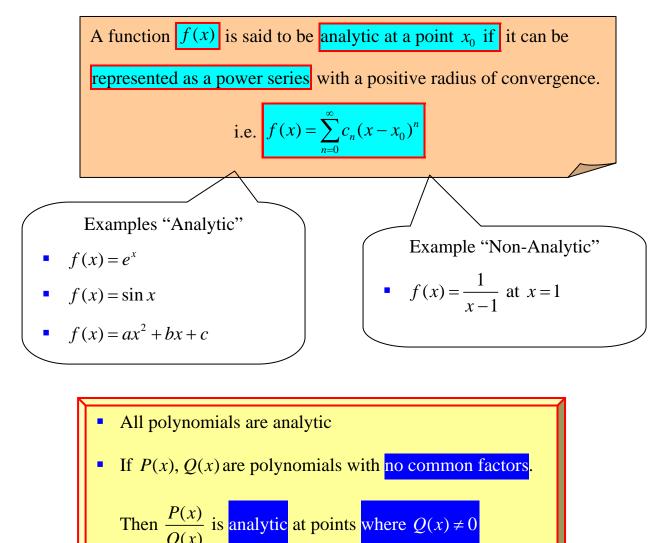


Both have radius of convergence R.



See Examples 2, 3, 4 done in class

- Not every function can be expressed as power series.
- Those which can be expressed as power series are given a special name.



# **Taylor Series**

Gives a way of finding power series of analytic functions

If 
$$f(x)$$
 is analytic at  $x_0$  then  

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!} (x - x_0)^n$$

$$= f(x_0) + f'(x_0) (x - x_0) + \frac{f''(x_0)}{2!} (x - x_0)^2 + \dots + \frac{f^{(n)}(x_0)}{n!} (x - x_0)^n + \dots$$

# Important basic series

Series	Interval of
	convergence
$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \dots$	-1 < x < 1
$e^{x} = \sum_{n=0}^{\infty} \frac{x^{n}}{n!} = 1 + x + \frac{x^{2}}{2!} + \frac{x^{3}}{3!} + \dots$	$-\infty < x < \infty$
$\sin x = \sum_{k=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$	$-\infty < x < \infty$
$\cos x = \sum_{k=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$	$-\infty < x < \infty$
$\tan^{-1} x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{2k+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$	$-1 \le x \le 1$

End of 6.1.1 Do Qs. 1-14

6.1.1<sub>6</sub>