Math 260 – Quiz # 8c

Name: Solution Sec.#: ________ Sr #: ______________

Solve the system:

$$\frac{dx}{dt} = -4x + y + z$$
$$\frac{dy}{dt} = x + 5y - z$$
$$\frac{dz}{dt} = y - 3z$$

$$\Rightarrow X' = \begin{bmatrix} -4 & 1 & 1 \\ 1 & 5 & -1 \\ 0 & 1 & -3 \end{bmatrix} X$$

$$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} -4-\lambda & 1 & 1 \\ 1 & 5-\lambda & -1 \\ 0 & 1 & -3-\lambda \end{vmatrix} = (-4-\lambda)\left[(5-\lambda)(-3-\lambda)+1\right] - \left[(-3-\lambda)-1\right] = 0$$

$$= (-4-\lambda)\left[\lambda^2 - 2\lambda - 14\right] - (-4-\lambda) = 0$$
$$= (-4-\lambda)\left[\lambda^2 - 2\lambda - 14 - 1\right] = 0$$
$$= (-4-\lambda)(\lambda^2 - 2\lambda - 15) = 0$$
$$= (-4-\lambda)(\lambda+3)(\lambda-5) = 0 \Rightarrow \lambda = -3, -4, 5$$

eigenvalues

$\underline{\lambda = -3}$ $(A + 3I)K = 0$

$$\begin{bmatrix} -1 & 1 & 1 \\ 1 & 8 & -1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving this system we get $k_2 = 0$, $k_1 = k_3 \Rightarrow K_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \Rightarrow X_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} e^{-3t}$

$\underline{\lambda = -4}$ $(A + 4I)K = 0$

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 9 & -1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \longmapsto \left[\begin{array}{ccc|c} 1 & 0 & -10 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{cases} k_1 = 10k_3 \\ k_2 = -k_3 \end{cases}$$

Take $k_3 = 1$ $\Rightarrow K_2 = \begin{bmatrix} 10 \\ -1 \\ 1 \end{bmatrix}$

$$\Rightarrow X_2 = \begin{bmatrix} 10 \\ -1 \\ 1 \end{bmatrix} e^{-4t}$$

$\underline{\lambda = 5}$ $(A - 5I)K = 0$

$$\begin{bmatrix} -9 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & -8 \end{bmatrix}\begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \longmapsto \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -8 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{cases} k_2 = 8k_3 \\ k_1 = k_3 \end{cases}$$

Take $k_3 = 1$ $\Rightarrow K_3 = \begin{bmatrix} 1 \\ 8 \\ 1 \end{bmatrix}$

$$\Rightarrow X_3 = \begin{bmatrix} 1 \\ 8 \\ 1 \end{bmatrix} e^{5t}$$

The general solution is

$$X = c_1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} e^{-3t} + c_2 \begin{bmatrix} 10 \\ -1 \\ 1 \end{bmatrix} e^{-4t} + c_3 \begin{bmatrix} 1 \\ 8 \\ 1 \end{bmatrix} e^{5t}$$