Math 260 – Quiz # 8

Name: Solution Sec.#:_____ Sr #:_____

Solve the system:

$$\frac{dx}{dt} = 2x + y + 6z$$
$$\frac{dy}{dt} = 2y + 5z$$
$$\frac{dz}{dt} = 2z$$

$$\Rightarrow X' = \begin{bmatrix} 2 & 1 & 6 \\ 0 & 2 & 5 \\ 0 & 0 & 2 \end{bmatrix} X$$

$$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} 2-\lambda & 1 & 6 \\ 0 & 2-\lambda & 5 \\ 0 & 0 & 2-\lambda \end{vmatrix} = (2-\lambda)^3 = 0 \Rightarrow \lambda = 2, 2, 2$$

$\Rightarrow \lambda = 2$ is an eigenvalue of multiplicity 3.

$$(A-2I)K_1 = 0 \Rightarrow \begin{bmatrix} 0 & 1 & 6 \\ 0 & 0 & 5 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow k_3 = 0, k_2 = 0, k_1 = 1$$

$$K_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} e^{2t}$$

$$(A-2I)K_2 = K_1 \Rightarrow \begin{bmatrix} 0 & 1 & 6 \\ 0 & 0 & 5 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow k_3 = 0, k_2 = 1, k_1 = 0$$

$$K_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \Rightarrow X_2 = \underbrace{\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} t + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right) e^{2t}}_{\sim (K_1 t + K_2) e^{\lambda t}}$$

$$(A-2I)K_3 = K_2 \Rightarrow \begin{bmatrix} 0 & 1 & 6 \\ 0 & 0 & 5 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \Rightarrow k_3 = \frac{1}{5}, k_2 = -\frac{6}{5}, k_1 = 0$$

$$K_3 = \begin{bmatrix} 0 \\ -\frac{6}{5} \\ \frac{1}{5} \end{bmatrix} \Rightarrow X_3 = \left(K_1 \frac{t^2}{2} + K_2 t + K_3 \right) e^{\lambda t} = \left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \frac{t^2}{2} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} t + \begin{bmatrix} 0 \\ -\frac{6}{5} \\ \frac{1}{5} \end{bmatrix} \right) e^{2t}$$

The general solution of the system is

$$X = C_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} e^{2t} + C_2 \left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} t + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right) e^{2t} + C_3 \left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \frac{t^2}{2} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} t + \begin{bmatrix} 0 \\ -\frac{6}{5} \\ \frac{1}{5} \end{bmatrix} \right) e^{2t}$$