Math 260 – Quiz # 6d

Name: Solution Sec.#: ______ Sr #: ____________

Use the method of undetermined coefficients to solve $y''' + y'' = e^x \cos x$

First, we solve the ass. hom. DE: $y''' + y'' = 0$

$\lambda^3 + \lambda^2 = 0 \Rightarrow \lambda^2(\lambda+1) = 0 \Rightarrow \lambda = 0, 0, -1$

$y_h = c_1 + c_2 x + c_3 e^{-x}$

$y_p = Ae^x \cos x + Be^x \sin x$ [no duplication] ✓

$y_p' = -Ae^x \sin x + Ae^x \cos x + Be^x \cos x + Be^x \sin x$

$= (-A+B)e^x \sin x + (A+B)e^x \cos x$

$y_p'' = (-A+B)e^x \cos x + (-A+B)e^x \sin x - (A+B)e^x \sin x + (A+B)e^x \cos x$

$= -2Ae^x \sin x + 2Be^x \cos x$

$y_p''' = -2Ae^x \cos x - 2Ae^x \sin x - 2Be^x \sin x + 2Be^x \cos x$

$= (-2A-2B)e^x \sin x + (-2A+2B)e^x \cos x$

Substitute in the given DE: $y_p''' + y_p'' = e^x \cos x$

$\Rightarrow (-2A-2B)e^x \sin x + (-2A+2B)e^x \cos x - 2Ae^x \sin x + 2Be^x \cos x = e^x \cos x$

$(-2A+4B)e^x \cos x + (-4A-2B)e^x \sin x = e^x \cos x$

$\Rightarrow \begin{cases} -2A+4B = 1 \\ -4A-2B = 0 \end{cases} \Rightarrow \boxed{A = -\frac{1}{10}}, \boxed{B = \frac{1}{5}}$

$\therefore y_p = -\frac{1}{10}e^x \cos x + \frac{1}{5}e^x \sin x$

The general solution is $y = y_h + y_p$

$= c_1 + c_2 x + c_3 e^{-x} - \frac{1}{10}e^x \cos x + \frac{1}{5}e^x \sin x$