

Math 260 - Quiz # 6c

Name: Solution

Sec. #: _____

Sr #: _____

Use the method of undetermined coefficients to solve $y'' - 6y' + 9y = 6x^2 + 2 - 12e^{3x}$

First, we solve the *hom.* DE: $y'' - 6y' + 9y = 0$

$$\lambda^2 - 6\lambda + 9 = 0 \Rightarrow (\lambda - 3)^2 = 0 \Rightarrow \lambda = 3, 3$$

$$y_h = c_1 e^{3x} + c_2 x e^{3x}$$

Initial assumption for y_p is $Ax^2 + Bx + C + Ee^{3x}$

But the last term duplicates with y_h , so we multiply ^{it} by x^2 to get:

$$y_p = Ax^2 + Bx + C + Ex^2 e^{3x}$$

$$y'_p = 2Ax + B + 3Ex^2 e^{3x} + 2Ex e^{3x}$$

$$y''_p = 2A + 9Ex^2 e^{3x} + 6Ex e^{3x} + 6Ex e^{3x} + 2Ee^{3x}$$

$$= 2A + 9Ex^2 e^{3x} + 12Ex e^{3x} + 2Ee^{3x}$$

Substitute in the given DE: $y''_p - 6y'_p + 9y_p = 6x^2 + 2 - 12e^{3x}$

$$\Rightarrow 2A + 9Ex^2 e^{3x} + 12Ex e^{3x} + 2Ee^{3x} - 6(2Ax + B + 3Ex^2 e^{3x} + 2Ex e^{3x}) + 9(Ax^2 + Bx + C + Ex^2 e^{3x})$$

$$= 6x^2 + 2 - 12e^{3x}$$

$$\Rightarrow 9Ax^2 + (-12A + 9B)x + (2A - 6B + 9C) + 2Ee^{3x} = 6x^2 + 2 - 12e^{3x}$$

Equating Coeff. $\Rightarrow 9A = 6 \Rightarrow \boxed{A = \frac{2}{3}}$, $-12A + 9B = 0 \Rightarrow \boxed{B = \frac{8}{9}}$, $2A - 6B + 9C = 2$, $2E = -12 \Rightarrow \boxed{C = \frac{2}{3}} \Rightarrow \boxed{E = -6}$

$$\therefore y_p = \frac{2}{3}x^2 + \frac{8}{9}x + \frac{2}{3} - 6x^2 e^{3x}$$

The general solution is: $y = y_h + y_p$

$$= c_1 e^{3x} + c_2 x e^{3x} + \frac{2}{3}x^2 + \frac{8}{9}x + \frac{2}{3} - 6x^2 e^{3x}$$