Math 260 – Quiz # 6b

Name: Solution Sec.#:______ Sr #:______

Use variation of parameters to solve $y'' + 4y = \tan 2x$

First, we solve the ass. hom. DE: $y'' + 4y = 0$

$\lambda^2 + 4 = 0 \Rightarrow \lambda = \pm 2i$

$y_h = c_1 \cos 2x + c_2 \sin 2x$. So, $y_1 = \cos 2x$, $y_2 = \sin 2x$

$y_p = u_1 \cos 2x + u_2 \sin 2x$

$$W = \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix} = 2\cos^2 2x + 2\sin^2 2x = 2$$

$$W_1 = \begin{vmatrix} 0 & \sin 2x \\ \tan 2x & 2\cos 2x \end{vmatrix} = -\sin 2x \tan 2x$$

$$W_2 = \begin{vmatrix} \cos 2x & 0 \\ -2\sin 2x & \tan 2x \end{vmatrix} = \cos 2x \tan 2x = \sin 2x$$

$$u_1' = \frac{W_1}{W} = -\frac{\sin 2x \tan 2x}{2}$$

$$u_2' = \frac{W_2}{W} = \frac{\sin 2x}{2}$$

$$u_1 = -\frac{1}{2}\int \sin 2x \tan 2x \, dx = -\frac{1}{2}\int \frac{\sin^2 2x}{\cos 2x} dx = -\frac{1}{2}\int \frac{1-\cos^2 2x}{\cos 2x} dx = \frac{1}{2}\int (\cos 2x - \sec 2x) dx$$

$$= \frac{1}{4}\sin 2x - \frac{1}{4}\ln|\sec 2x + \tan 2x|$$

$$u_2 = \frac{1}{2}\int \sin 2x \, dx = -\frac{1}{4}\cos 2x$$

$$y_p = u_1 \cos 2x + u_2 \sin 2x = \left(\frac{1}{4}\sin 2x - \frac{1}{4}\ln|\sec 2x + \tan 2x|\right)\cos 2x - \frac{1}{4}\cos 2x \sin 2x$$

$$= -\frac{1}{4}\cos 2x \ln|\sec 2x + \tan 2x|$$

$$y = y_h + y_p = c_1 \cos 2x + c_2 \sin 2x - \frac{1}{4}\cos 2x \ln|\sec 2x + \tan 2x|$$