MATH 202 - (082)

Review for Exam 1

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1. (a) For each of the following, state whether the equation is linear or nonlinear, and give its order.

Equation	Linearity	Order
$y''' - 3y' + 2y = 0$		
$x(y'')^3 + (y')^4 - y = 0$		
$y' = 1 - xy + y^2$		

(b) Identify the following 1st Order ODE as **Separable**, **Linear in *y*** (or **in *x***), **Homogeneous** (**with its degree**), **Bernoulli**, or **Exact**. Also write the ODE in the **standard form** of the identified category. (Write "**None**" if the ODE is not in any of the above-mentioned categories).

i. $(3y^2 + y + 3x)dx = (4 - 6xy - x)dy$

ii. $3\dfrac{dy}{dx} = 4x - y$

iii. $(e^{y/x} + e^{x^3/y^3} + 1)dy = (1 + \ln(y/x))dx$

iv. $(y + y^2)dx - (x + x^2)dy = 0$

2. Find a region in the (x, y) plane for which the DE: $(y - x)y' = y + x$ would have a unique solution through a point (x_0, y_0) in the region.

3. It is known that $y = \dfrac{1 + ce^{2x}}{1 + ce^{-2x}}$ is a one parameter family of Solutions of the ODE $y' = y^2 - 1$. Find a Singular Solution of this ODE.

4. Show that $\dfrac{1}{e^{x^2}} + \dfrac{1}{y^2} = 2$ is a solution of the differential equation $e^{x^2}\dfrac{dy}{dx} + xy^3 = 0$

5. Show that the differential equation $6x^2dy - y^3(2y + \frac{x}{y^2})dx = 0$ is Bernoulli equation, and hence find its particular solution subject to condition $y(1) = -1$.

6. Determine whether the following set of functions is linearly independent on $(-\infty, \infty)$.

$$f_1(x) = 4x - 3x^2, \quad f_2(x) = x^2, \quad f_3(x) = x.$$

7. Determine, without solving, whether the following DE possesses a unique solution through the point (2,-3): $\dfrac{dy}{dx} = \sqrt{y^2 - 9}$

8. Use appropriate substitution in order to convert the following ODE to Separable, and then find its solution: $(x^2 + y^2)dx - xydy = 0$

9. Solve the following differential equation: $\frac{dy}{dx} = \frac{y^2 + xy^2}{x^2 y - x^2}$

10. Solve the following differential equation: $(3x^2y + y^2)dx + (x^3 + 2xy)dy = 0$

11. A thermometer is taken from an inside room to outside, where the air temperature is $5^{\circ}F$. After 1 minute the thermometer reads $55^{\circ}F$, and after 5 minutes it reads $30^{\circ}F$. What is the initial temperature of the inside room?

12. Consider the differential equation $y'' - y = x$.

(a) Determine the largest interval for which a unique solution to the initial value Problem $y'' - y = x$; $y(0) = 1$, $y'(0) = 0$ is guaranteed.

(b) Write the corresponding homogeneous equation and show that $y_1 = e^x$, $y_2 = e^{-x}$ form a fundamental set of solutions to the homogeneous equation and give the general solution to this homogeneous equation.