Math 202 Quiz # 5d

Name: Solution Section # ____________ Serial # ____

Find the general solution of the DE: $y^{(4)} - K^2 y'' = 1$ ——— (1)

First solve the ass. hom. DE: $y^{(4)} - K^2 y'' = 0$

$\lambda^4 - K^2\lambda^2 = 0$

$\lambda^2[\lambda^2 - K^2] = 0 \Rightarrow \lambda = 0, 0, K, -K$

$\boxed{y_H = c_1 + c_2 x + c_3 e^{Kx} + c_4 e^{-Kx}}$ ——— (2)

write the DE (1) as:

$(D^4 - K^2 D^2) y = 1$

Note that $Ann(1) = D$

$\therefore D(D^4 - K^2 D^2) y = 0$. Now we solve this DE:

$\lambda(\lambda^4 - K^2\lambda^2) = 0$

$\lambda^3(\lambda^2 - K^2) = 0 \Rightarrow \lambda = 0, 0, 0, K, -K$

$y = c_1 + c_2 x + c_3 x^2 + c_4 e^{Kx} + c_5 e^{-Kx}$ ——— (3)

Compare (3) & (2) to get:

$\boxed{y_p = Ax^2}$

$\Rightarrow y_p' = 2Ax, \quad y_p'' = 2A, \quad y_p''' = y_p^{(4)} = 0$

substitute in (1):

$0 - K^2(2A) = 1 \Rightarrow A = -\frac{1}{2K^2}$

$\therefore y_p = -\frac{1}{2K^2} x^2$

The solution of (1) is:

$$\boxed{\begin{aligned} y &= y_H + y_p \\ &= c_1 + c_2 x + c_3 e^{Kx} + c_4 e^{-Kx} - \frac{1}{2K^2} x^2 \end{aligned}}$$