Math 202 Quiz # 5a

Name: Solution Sr. # ____________ Section # ____

1. Find a linear differential operator that annihilates the function: $f(x) = (7 - e^x)^2$.

$f(x) = 49 - 14e^x + e^{2x}$

$D \quad D-1 \quad D-2$

$Ann[f(x)] = D(D-1)(D-2)$

2. Solve the following DE by undetermined coefficients: $y'' + 4y = 4\cos x + 3\sin x - 8$

First solve the associated hom DE: $y'' + 4y = 0$

$\lambda^2 + 4 = 0 \Rightarrow \lambda = \pm 2i$

$y_H = c_1 \cos 2x + c_2 \sin 2x$ - - - - - - - - (1)

Write the given DE as $(D^2+4)y = 4\cos x + 3\sin x - 8$

$Ann(R.H.S.) = D(D^2+1)$

$D(D^2+1)(D^2+4)y = D(D^2+1)(4\cos x + 3\sin x - 8)$

$D(D^2+1)(D^2+4)y = 0$

Solving this equation $\Rightarrow \lambda(\lambda^2+1)(\lambda^2+4) = 0 \Rightarrow \lambda = 0, \pm i, \pm 2i$

The solution is $y = c_1 + c_2\cos x + c_3 \sin x + \underbrace{c_4 \cos 2x + c_5 \sin 2x}_{y_H}$ ---(2)

Comparing (1) & (2), we get

$y_p = A + B\cos x + C\sin x \Rightarrow y_p' = -B\sin x + C\cos x \Rightarrow y_p'' = -B\cos x - C\sin x$

Substitute in the given DE: $y_p'' + 4y_p = 4\cos x + 3\sin x - 8$

$-B\cos x - C\sin x + 4A + 4B\cos x + 4C\sin x = 4\cos x + 3\sin x - 8$

$3B\cos x + 3C\sin x + 4A = 4\cos x + 3\sin x - 8$

Equating coeffs $\Rightarrow 4A = -8 \Rightarrow \boxed{A = -2}$, $3B = 4 \Rightarrow \boxed{B = \frac{4}{3}}$, $3C = 3 \Rightarrow \boxed{C = 1}$

$\therefore y_p = -2 + \frac{4}{3}\cos x + \sin x$

The solution is $y = y_H + y_p$

$= c_1 \cos 2x + c_2 \sin 2x + \frac{4}{3}\cos x + \sin x - 2$