Math 202 Quiz # 8d

Name: Solution Sec. # ____________ Ser. # ____

Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} -1 & 2 \\ -1 & -3 \end{bmatrix}$

$$|A - \lambda I| = 0 \quad \Rightarrow \quad \begin{vmatrix} -1-\lambda & 2 \\ -1 & -3-\lambda \end{vmatrix} = 0$$

$$(-1-\lambda)(-3-\lambda) + 2 = 0$$

$$\lambda^2 + 4\lambda + 5 = 0 \quad \Rightarrow \quad \lambda = -2 \pm i$$, the eigenvalues of A.

To find the eigen vectors:

$\lambda = -2+i$

$$[A - (-2+i)]K = 0$$

$$\begin{bmatrix} 1-i & 2 \\ -1 & -1-i \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 1-i & 2 & 0 \\ -1 & -1-i & 0 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & 1+i & 0 \\ 1-i & 2 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & 1+i & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow k_1 = -(1+i)k_2$$

Take $k_2 = -1$

$\Rightarrow k_1 = 1+i$

$\therefore K_1 = \begin{bmatrix} 1+i \\ -1 \end{bmatrix}$ is an eigenvector corresponds to $\lambda = -2+i$

We can conclude that $K_2 = \begin{bmatrix} 1-i \\ -1 \end{bmatrix}$ is an eigenvector corresponds to $\lambda = -2-i$.