Math 202 Quiz # 8

Name: ___Solution___ Sec. # ________ Ser. # ____

Find the eigenvalues and eigenvectores of the matrix $A = \begin{bmatrix} -1 & -1 & 0 \\ -2 & 2 & -1 \\ 0 & -2 & -1 \end{bmatrix}$

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} -1-\lambda & -1 & 0 \\ -2 & 2-\lambda & -1 \\ 0 & -2 & -1-\lambda \end{vmatrix} = 0$$

$$(-1-\lambda)\left[(2-\lambda)(-1-\lambda)-2\right] + \left[-2(-1-\lambda)\right] = 0$$

$$= - - - - - - - - -$$

$$= (\lambda+1)(\lambda-3)(\lambda+2) = 0 \quad \Rightarrow \lambda = -1, -2, 3,$$ the eigenvalues of A.

$\underline{\underline{\lambda = -1}}$ $(A - \lambda I)K = 0$

$$\Rightarrow \left[\begin{array}{ccc|c} 0 & -1 & 0 & 0 \\ -2 & 3 & -1 & 0 \\ 0 & -2 & 0 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} -2 & 3 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & -\frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow k_2 = 0,\ k_1 = -\tfrac{1}{2}k_3$$

$\therefore K_1 = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$, the eigen vector corresponds to $\lambda = -1$.

$\underline{\underline{\lambda = -2}}$ $(A - \lambda I)K = 0$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ -2 & 4 & -1 & 0 \\ 0 & -2 & 1 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & -2 & 1 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow k_2 = \tfrac{1}{2}k_3,\ k_1 = k_2$$

$\therefore K_2 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$, the eigen vector corresponds to $\lambda = -2$.

$\underline{\underline{\lambda = 3}}$ $(A - \lambda I)K = 0$

$$\left[\begin{array}{ccc|c} -4 & -1 & 0 & 0 \\ -2 & -1 & -1 & 0 \\ 0 & -2 & -4 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & \frac{1}{4} & 0 & 0 \\ -2 & -1 & -1 & 0 \\ 0 & -2 & -4 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & \frac{1}{4} & 0 & 0 \\ 0 & -\frac{1}{2} & -1 & 0 \\ 0 & -2 & -4 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & \frac{1}{4} & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -2 & -4 & 0 \end{array}\right] \Rightarrow \left.\begin{array}{l} k_2 = -2k_3 \\ k_1 = -\frac{k_2}{4} \end{array}\right\}$$

$\therefore K_3 = \begin{bmatrix} \frac{1}{2} \\ -2 \\ 1 \end{bmatrix}$, the eigenvector corresponds to $\lambda = 3$.