Math 202 Quiz # 4c

Name: Solution Sr. # __________ Section # ____

Given that $y_1 = x\cos(\ln x)$ is a solution of the DE: $x^2y'' - xy' + 2y = 0,\ x \neq 0.$

1. Find a second linearly independent solution.
2. Check your answer by verifying that the two solutions form a fundamental set of solutions.
3. Write the general solution of the DE.

1- First write the DE as: $y'' - \frac{1}{x}y' + \frac{2}{x^2}y = 0$

$P(x) = \frac{1}{x}$

Use the formula:

$$y_2 = y_1 \int \frac{e^{-\int P dx}}{y_1^2} dx$$

$$= x\cos(\ln x) \int \frac{e^{-\int \frac{1}{x} dx}}{x^2\cos^2(\ln x)} dx = x\cos(\ln x) \int \frac{\frac{1}{x}}{x^2\cos^2(\ln x)} dx$$

$$= x\cos(\ln x) \int \frac{1}{x\cos^2(\ln x)} dx = x\cos(\ln x) \int \frac{\sec^2(\ln x)}{x} dx$$

$$= x\cos(\ln x)\tan(\ln x)$$

$$\therefore\ y_2 = x\sin(\ln x)$$

2- $$W(y_1, y_2) = W\big(x\cos(\ln x), x\sin(\ln x)\big) = \begin{vmatrix} x\cos(\ln x) & x\sin(\ln x) \\ -\sin(\ln x) + \cos(\ln x) & \cos(\ln x) + \sin(\ln x) \end{vmatrix}$$

$$= x \neq 0$$

$\Rightarrow y_1, y_2$ are linearly indep.

$\Rightarrow \{y_1, y_2\}$ is a fundamental set of solutions for the DE.

3- The general solution is $y = c_1 x\cos(\ln x) + c_2 x\sin(\ln x)$