Math 202 Quiz # 4b

Name: Solution Sr. # ___________ Section # ____

Given that $y_1 = \dfrac{\cos x}{\sqrt{x}}$ is a solution of the DE: $x^2 y'' + xy' + \left(x^2 - \dfrac{1}{4}\right)y = 0$ on $(0, \dfrac{\pi}{2})$.

1. Find a second linearly independent solution.
2. Check your answer by verifying that the two solutions form a fundamental set of solutions.
3. Write the general solution of the DE.

1- First write the DE as: $y'' + \frac{1}{x} y' + \left(1 - \frac{1}{4x^2}\right) y = 0$

$P(x) = \frac{1}{x}$

use the formula:

$$y_2 = y_1 \int \frac{e^{-\int P dx}}{y_1^2} dx$$

$$= \frac{\cos x}{\sqrt{x}} \int \frac{e^{-\int \frac{1}{x} dx}}{\left[\frac{\cos x}{\sqrt{x}}\right]^2} dx = \frac{\cos x}{\sqrt{x}} \int \frac{\frac{1}{x}}{\frac{\cos^2 x}{x}} dx$$

$$= \frac{\cos x}{\sqrt{x}} \int \frac{1}{\cos^2 x} dx = \frac{\cos x}{\sqrt{x}} \int \sec^2 x \, dx = \frac{\cos x}{\sqrt{x}} \tan x$$

$$\therefore y_2 = \frac{\sin x}{\sqrt{x}}$$

2- $$W(y_1, y_2) = W\left(\frac{\cos x}{\sqrt{x}}, \frac{\sin x}{\sqrt{x}}\right) = \begin{vmatrix} \frac{\cos x}{\sqrt{x}} & \frac{\sin x}{\sqrt{x}} \\ -\sqrt{x} \sin x - \frac{\cos x}{2\sqrt{x}} & \sqrt{x} \cos x - \frac{\sin x}{2\sqrt{x}} \end{vmatrix}$$

$$= \frac{1}{2x} \neq 0$$

$\Rightarrow y_1, y_2$ are linearly indep.

$\Rightarrow \{y_1, y_2\}$ is a fundamental set of solutions for the DE.

3- The general solution is $y = c_1 \frac{\cos x}{\sqrt{x}} + c_2 \frac{\sin x}{\sqrt{x}}$