

Name: Solution I.D. # _____ Ser. # _____

1. In each of the following DE determine if it is separable, linear or none:

(a) $y^3 \frac{dy}{dx} = (y^4 + 1) \cos x$

$$\frac{dy}{dx} = \frac{y^4 + 1}{y^3} \cos x = g(y) f(x) \implies \text{Separable}$$

(b) $(x^2 - 4x - 3xy + 4)dx = (x^2 - x - 2)dy$

$$\frac{dy}{dx} = \frac{x^2 - 4x - 3xy + 4}{x^2 - x - 2} \implies \frac{dy}{dx} - \frac{3x}{x^2 - x - 2} y + \frac{x^2 - 4x + 4}{x^2 - x - 2} = 0 \implies \text{Linear}$$

(c) $(\cos x)y''' - (\sin x)y' - 8 = 0$

Linear

2. Solve the following initial value problem: $y' = y \cos x - xy$, $y(\pi) = 1$

$$\frac{dy}{dx} = y(\cos x - x)$$

$$\frac{dy}{y} = (\cos x - x) dx$$

Integrate to get:

$$\ln|y| = \sin x - \frac{x^2}{2} + C \quad \text{or} \quad \ln y = \sin x - \frac{x^2}{2} + C$$

Use the initial condition $y(\pi) = 1 \implies C = \frac{\pi^2}{2}$

$$\therefore \text{the solution is } \ln y = \sin x - \frac{x^2}{2} + \frac{\pi^2}{2}$$

$$y = e^{\sin x + \frac{\pi^2 - x^2}{2}}$$