

Name: Solution

Math 260 - Quiz # 5

Sr #: _____

Find the inverse, if it exists, for each of the following matrices:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 5 & 5 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 & -3 \\ 1 & -2 & 1 \\ 5 & -2 & -3 \end{bmatrix}$$

$$[A | I] = \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 3 & 0 & 1 & 0 \\ 5 & 5 & 1 & 0 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{-5R_1 + R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 3 & 0 & 1 & 0 \\ 0 & 0 & -4 & -5 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{\frac{1}{2} R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & \frac{3}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & -4 & -5 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{-\frac{1}{4} R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & \frac{3}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & \frac{5}{4} & 0 & -\frac{1}{4} \end{array} \right]$$

$$\xrightarrow{\begin{array}{l} -\frac{3}{2} R_3 + R_2 \\ -R_3 + R_1 \end{array}} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & -\frac{1}{4} & 0 & \frac{1}{4} \\ 0 & 1 & 0 & -\frac{15}{8} & \frac{1}{2} & \frac{7}{8} \\ 0 & 0 & 1 & \frac{5}{4} & 0 & -\frac{1}{4} \end{array} \right]$$

$$\xrightarrow{-R_2 + R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{13}{8} & -\frac{1}{2} & -\frac{1}{8} \\ 0 & 1 & 0 & -\frac{15}{8} & \frac{1}{2} & \frac{7}{8} \\ 0 & 0 & 1 & \frac{5}{4} & 0 & -\frac{1}{4} \end{array} \right] = [I | A^{-1}]$$

$$\therefore A^{-1} = \begin{bmatrix} \frac{13}{8} & -\frac{1}{2} & -\frac{1}{8} \\ -\frac{15}{8} & \frac{1}{2} & \frac{7}{8} \\ \frac{5}{4} & 0 & -\frac{1}{4} \end{bmatrix}$$

Cont.?

Next, for the matrix B , we have:

$$[B | I] = \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 & 1 & 0 \\ 5 & -2 & -3 & 0 & 0 & 1 \end{array} \right]$$

$$\begin{array}{l} -R_1 + R_2 \\ -5R_1 + R_3 \end{array} \rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & -4 & 4 & -1 & 1 & 0 \\ 0 & -12 & 12 & -5 & 0 & 1 \end{array} \right]$$

$$-\frac{1}{4}R_2 \rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & 1 & -1 & \frac{1}{4} & -\frac{1}{4} & 0 \\ 0 & -12 & 12 & -5 & 0 & 1 \end{array} \right]$$

$$12R_2 + R_3 \rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & 1 & -1 & \frac{1}{4} & -\frac{1}{4} & 0 \\ 0 & 0 & 0 & -2 & -3 & 1 \end{array} \right]$$

$$\Rightarrow B \sim C \quad \text{where } C = \left[\begin{array}{ccc} 1 & 2 & -3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array} \right]$$

and C has a row of zeros.

Hence B is a singular matrix.